THE SINGULAR SURFACE IN THE PARAMETER SPACE; APPLICATIONS

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THESIS

THE SINGULAR SURFACE IN THE PARAMETER SPACE; APPLICATIONS

by

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Compensation techniques using the singular line are demonstrated and a computer program for determining the existance of singular lines is presented.

The Singular Surface in the Parameter Space; Applications

by

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I. INTRODUCTION

Compensation of linear control systems by the classical methods cf root locus and frequency response is adequate when there is a single variable. However when there are two or more variables compensation of physical systems for desired system dynamics becomes a complex task [1].

In modern control engineering systems characteristic equations with two or more unknown parameters result. In 1959, Mitrovic's Method [2] was developed to portray the relationship between the roots of the characteric equation and two cceefficients of the system. This method allowed the two lowest ordered coefficients of the characteristic equation to be varied. Siljak expanded this method to the Coefficient Plane to allow the variation of any two of the coefficients of the characteristic equation. For example:

$$f(s) = B_i s^i + B_j s^j + \sum_{\substack{k=0 \\ k \neq i,j}}^{n} a_k s^k \qquad (1-1)$$

where: a_k are real constants

B_i and B_j are real variables

Application of this method resulted in constant zeta, omega, and sigma curves on the B_i-B_j plane specifying the roots of equation (1-1) for any choice of ζ and ω_n.

In 1964, Siljak [3] extended Mitrovic's work so that

the two variable parameters may appear linearly in the coefficients of the characteristic equation. consider the linear case of the characteristic equation:

$$f(s) = \sum_{k=0}^{n} a_k s^k = 0 \qquad (1\text{-}2)$$

where: $$a_k = b_k \alpha + c_k \beta + d_k \qquad (1\text{-}3)$$

and: and are real variable parameters

Application of this method resulted in constant zeta, omega and sigma curves for any choice of α and β on the α-β plane specifying the roots of equation (1-2).

Siljak's Parameter Plane Method provides the designer with a simple procedure for factoring a characteristic equation and displaying the results in a parameter plane diagram. It allows the designer to obtain information about system stability and the affect of parameter variations and adjustments on stability. In the general case the Parameter Plane Method can be applied to any control problem in which it is necessary to determine how the variation of one or more parameters effect the root location of a system.

In 1967, Bowie [4] discovered that a limitation existed in parameter plane theory. While attempting to solve a sixth order characteristic equation, Bowie found that he could not find a complete set of roots in an area of the parameter plane that should have contained only real roots. By substituting the values of α and β into the characteristic equation he found the roots. It was discovered that a set of complex roots existed corresponding to a $\zeta - \omega_n$ pair in the left-half s-plane. Further

investigation showed that this ζ-ω_n pair formed a straight line in the parameter plane. This was contrary to the then existing parameter plane theory which stated that a ζ-ω_n pair will only occupy a point in the parameter plane.

Bowie called this line of constant ζ -constant ω_n the SINGULAR LINE as it was formed by a singular value of ζ and ω_n The singular line added a new dimension to the parameter plane because it allows the designer to hold one root location constant while moving others to achieve desired system characteristics by varying α and β.

II. THE SINGULAR LINE IN THE PARAMETER PLANE

A. THE PARAMETER PLANE

The Parameter Plane Method was developed by Siljak [3] as an extension to Mitrovic's Coefficient Plane. The parameter plane is a mapping of points in the s-plane into the α - β plane. The following is the mathematical development of that transformation.

Consider the characteristic equation:

$$f(s) = \sum_{k=0}^{n} a_k s^k \qquad (2\text{-}1)$$

where: a_k $(k=0,1,2,\ldots,n)$ are real variables

and:
$$s = -\sigma + j\omega = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \qquad (2\text{-}2)$$

Examination of figure 2.1 shows

$$-\zeta = \mathrm{Cos}\,\theta$$

and
$$\sqrt{1-\zeta^2} = \mathrm{Sin}\,\theta \qquad (2\text{-}3)$$

By substituting equation (2-3), equation (2-2) may be expressed as:

$$s = \omega_n\,(\mathrm{Cos}\,\theta + j\mathrm{Sin}\,\theta) \qquad (2\text{-}4)$$

and
$$s^k = \omega_n^k\,(\mathrm{Cos}\,k\theta + j\mathrm{Sin}\,k\theta) \qquad (2\text{-}5)$$

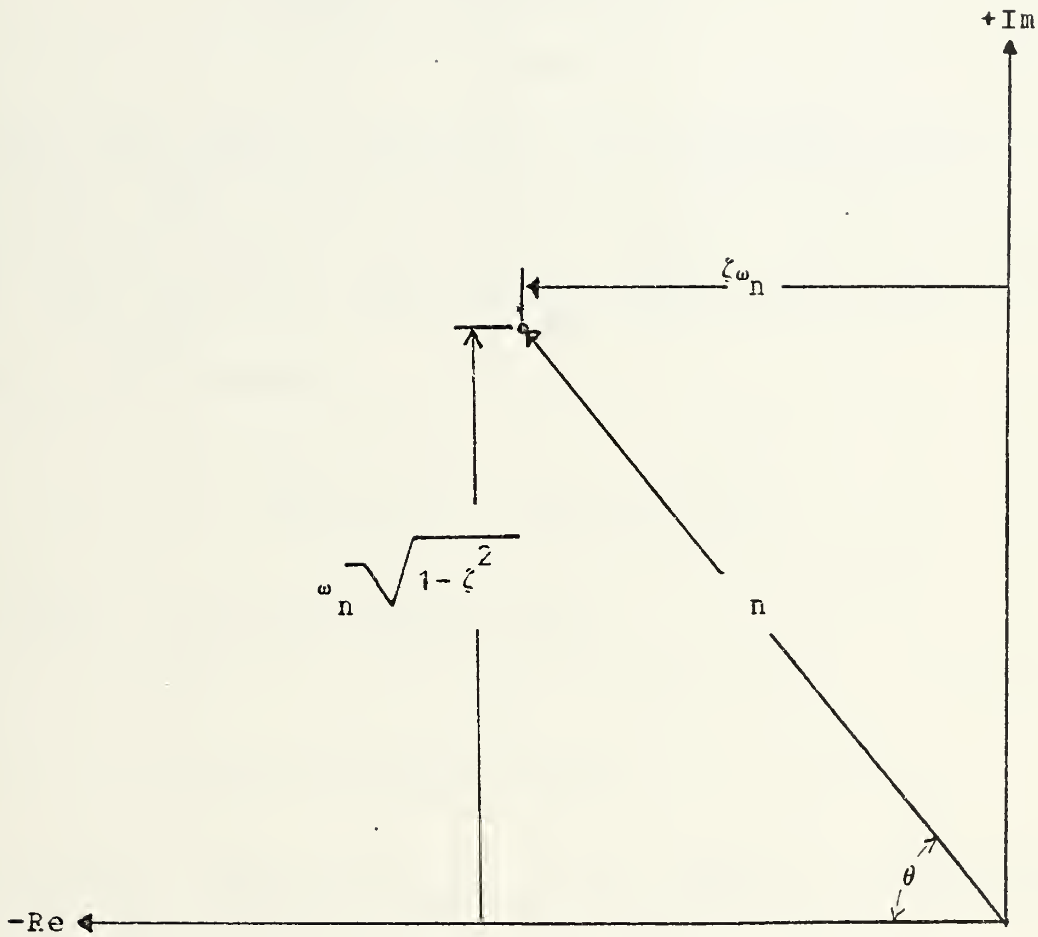


Figure 2.1
A POINT IN THE S-PLANE

Since

$$\theta = \mathrm{Cos}^{-1}(-\zeta) \qquad (2\text{-}6)$$

it follows that

$$\mathrm{Cos}k\theta = \mathrm{Cos}k[\mathrm{Cos}(-\zeta)] \qquad (2\text{-}7)$$

and

$$\mathrm{Sin}k\theta = \mathrm{Sin}k[\mathrm{Cos}(-\zeta)] \qquad (2\text{-}8)$$

Equation (2-7) can be expressed as the Chebyshev function of the first kind, $T_k(-\zeta)$ and similarly equation (2-8) can be expressed in terms of the Chebshev function of the second kind, $\mathrm{Sin}\ U_k[-\zeta]$. Substituting equations (2-7) and (2-8) equation (2-5) becomes

$$s^k = \omega_n^k\,[T_k(-\zeta) + j\mathrm{Sin}\ U_k(-\zeta)] \qquad (2\text{-}9)$$

where: $T_k(-\zeta)\zeta = (-1)^k\ T_k(\zeta)$

(2-10)

and $U_k(-\zeta) = (-1)^{k+1}\ U_k(\zeta)$

Substituting equation (2-10) into equation (2-9), then substituting equation (2-9) into equation (2-1), equation (2-1) becomes

$$f(s) = \sum_{k=0}^{n} a_k\,\omega_n^k\,(-1)^k\,T_k(\zeta) + j\sin\sum_{k=0}^{n} a_k\,\omega_n^k\,(-1)^{k+1}\,U_k(\zeta) = 0 \quad (2\text{-}11)$$

For any non-zero value of both the real and imaginary parts of equation (2-11) must go to zero independently therefore,

$$\sum_{k=0}^{n} a_k {}_n^k (-1)^k T_k(\zeta) = 0$$

(2-12)

$$\sum_{k=0}^{n} a_k {}_k (-1)^{k+1} U_k(\zeta) = 0$$

The Chebyshev functions of the first and second kind are given by the recursion formulae

$$T_{k+1}(\zeta) = 2T_k(\zeta) - T_{k-1}(\zeta)$$

(2-13)

$$U_{k+1}(\zeta) = 2U(\ ) - U_{k-1}(\zeta)$$

where: $T_0(\zeta) = 1$ $\quad U_{-1}(\zeta) = -1$

$T_1(\zeta) = \zeta$ $\quad U_0(\zeta) = 0$

$U_1(\zeta) = 1$

To eliminate $T_k(\zeta)$ from equation (2-12) it is noted that

$$T_k(\zeta) = U_k(\zeta) - U_{k-1}(\zeta) \tag{2-14}$$

then equation (2-12) may be rewritten

$$\sum_{k=0}^{n} (-1)^k a_k \omega_n^k [\ U_k(\zeta)\ ] - \sum_{k=0}^{n} (-1)^k a_k \omega_n^k U^{k-1}(\zeta) = 0 \tag{2-15a}$$

$$\sum_{k=0}^{n} (-1)^{k+1} a_k \omega_n^k U_k(\zeta) = 0 \tag{2-15b}$$

Examination of equation (2-15a) shows that its first part is

a linearly dependent solution to equation (2-15b) for all non-zero values of zeta therefore equation (2-15) becomes

$$\sum_{k=0}^{n} (-1)^k a_k \, {}_nU^k_{k-1}(\zeta) = 0$$

$$\sum_{k=0}^{n} (-1)^k a_k \, {}_nU^k_{k}(\zeta) = 0 \qquad (2\text{-}16)$$

For the linear case:

$$a_k = \alpha b_k + \beta c_k + d_k \qquad (2\text{-}17)$$

where: b_k, c_k, and d_k are real constants

and: α and β are the variable parameters

Substitution of equation (2-17) into equation (2-16) yields:

$$B_1\alpha + C_1\beta + D_1 = 0$$

$$(2\text{-}18)$$

and

$$B_2\alpha + C_2\beta + D_2 = 0$$

where:

$$B_1 = \sum_{k=0}^{n} (-1)^k b_k \, {}_nU^k_{k-1}(\zeta) \qquad B_2 = \sum_{k=0}^{n} (-1)^k b_k \, {}_nU^k_{k}(\zeta)$$

$$C_1 = \sum_{k=0}^{n} (-1)^k c_k \, {}_nU^k_{k-1}(\zeta) \qquad C_2 = \sum_{k=0}^{n} (-1)^k c_k \, {}_nU^k_{k}(\zeta) \qquad (2\text{-}19)$$

$$D_1 = \sum_{k=0}^{n} (-1)^k d_k U_{n\,k-1}^{k}(\zeta) \qquad D_2 = \sum_{k=0}^{n} (-1)^k d_k U_{n\,k}^{k}(\zeta)$$

Application of Cramer's Rule for the solution of simultanous linear equations to equation (2-18) yields the desired parameter plane solution equation:

$$\alpha = \frac{C_1 D_2 - C_2 D_1}{B_1 C_2 - B_2 C_1} \qquad \beta = \frac{B_2 D_1 - B_1 D_2}{B_1 C_2 - B_2 C_1}. \qquad (2\text{-}20)$$

Siljak [3] defined the parameter plane as a rectangular plot with as the abscissa and as the ordinate. Equation (2-20) gives α and β as a function of ω_n and ζ. By fixing $\omega = \omega_i$ and varying ω_n over the range from zero to infinity in equation (2-20), a curve is produced in the α-β plane for the fixed value of zeta. This curve specifies the α and β pairs which will cause equation (2-1) to have a pair of roots corresponding to the fixed value of zeta. Similarly by fixing $\omega_n = \omega_{ni}$ and varying zeta from -1 to +1 in equation (2-20), a curve is produced in the α-β plane for the fixed value of ω_{ni}. This curve specifies the α and β pairs which cause equation (2-1) to have a pair of roots corresponding to the fixed value of ω_n. Thus equation (2-20) allows the mapping of complex points in the s-plane to the parameter plane.

To transform the real axis points in the s-plane into the parameter plane, replaces s in equation (2-1) by $-\sigma$ to obtain:

$$f(s) = \sum_{k=0}^{n} a_k(-\sigma)^k = 0 \qquad (2\text{-}21)$$

Now equation (2-17) is substituted into equation (2-21) to obtain:

$$\sum_{k=0}^{n} (b_k\alpha + \beta c_k + d)(-\sigma)^k = 0 \qquad (2\text{-}22)$$

Simplifying equation (2-22) obtains:

$$B(\sigma) + \beta C(\sigma) + D(\sigma) = 0 \qquad (2\text{-}23)$$

where:

$$B(\sigma) = \sum_{k=0}^{n} (-1)^k b_k \sigma^k$$

$$C(\sigma) = \sum_{k=0}^{n} (-1)^k c_k \sigma^k \qquad (2\text{-}24)$$

$$D(\sigma) = \sum_{k=0}^{n} (-1)^k d_k \sigma^k$$

For a given value of σ the functions $B(\sigma)$, $C(\sigma)$, and $D(\sigma)$ are constants and equation (2-20) results in a straight line in the parameter plane with a locus of point corresponding to the real roots $s=-\sigma$.

A digital computer program PARAM A was written by Nutting [5] to aid in solving the characteristic equation and plotting the constant zeta, omega, found at the end of this thesis and a brief explanation is found in Appendix A.

EXAMPLE I:

Consider a system whose characteristic equation is the fourth order polynomial:

$$f(s)=s^4+8.5s^3+(5\alpha+28)s^2+(12.5\alpha+25\beta+42.5)s+50\beta+25=0 \qquad 2\text{-}25)$$

Figure 2.2 shows the parameter plane for the following values of zeta, omega, and sigma:

OMEGA	ZETA	SIGMA
3.0	0.00	0.5
4.0	0.30	1.0
5.0	0.35	2.0
6.0	0.40	3.0
7.0	0.45	4.0
8.0	0.50	5.0
9.0	1.00	6.0

Examination of figure 2.2 shows the region of stability bounded by $\zeta=0.0$, $\zeta=1.0$, and $\sigma=0.0$. All values of α and β inside this region will result in a stable system. The roots of the system can be determined by selecting a point in the plane and reading the corresponding values of zeta, omega, and sigma. Zeta and omega will form a complex of roots so that:

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

and the sigma point will form the real roots of the form:

$$s = -\sigma$$

Although it may not be immediatly obvious, there are two sigma points through each point in the region. Careful interpolation or the plotting of additionalines is required

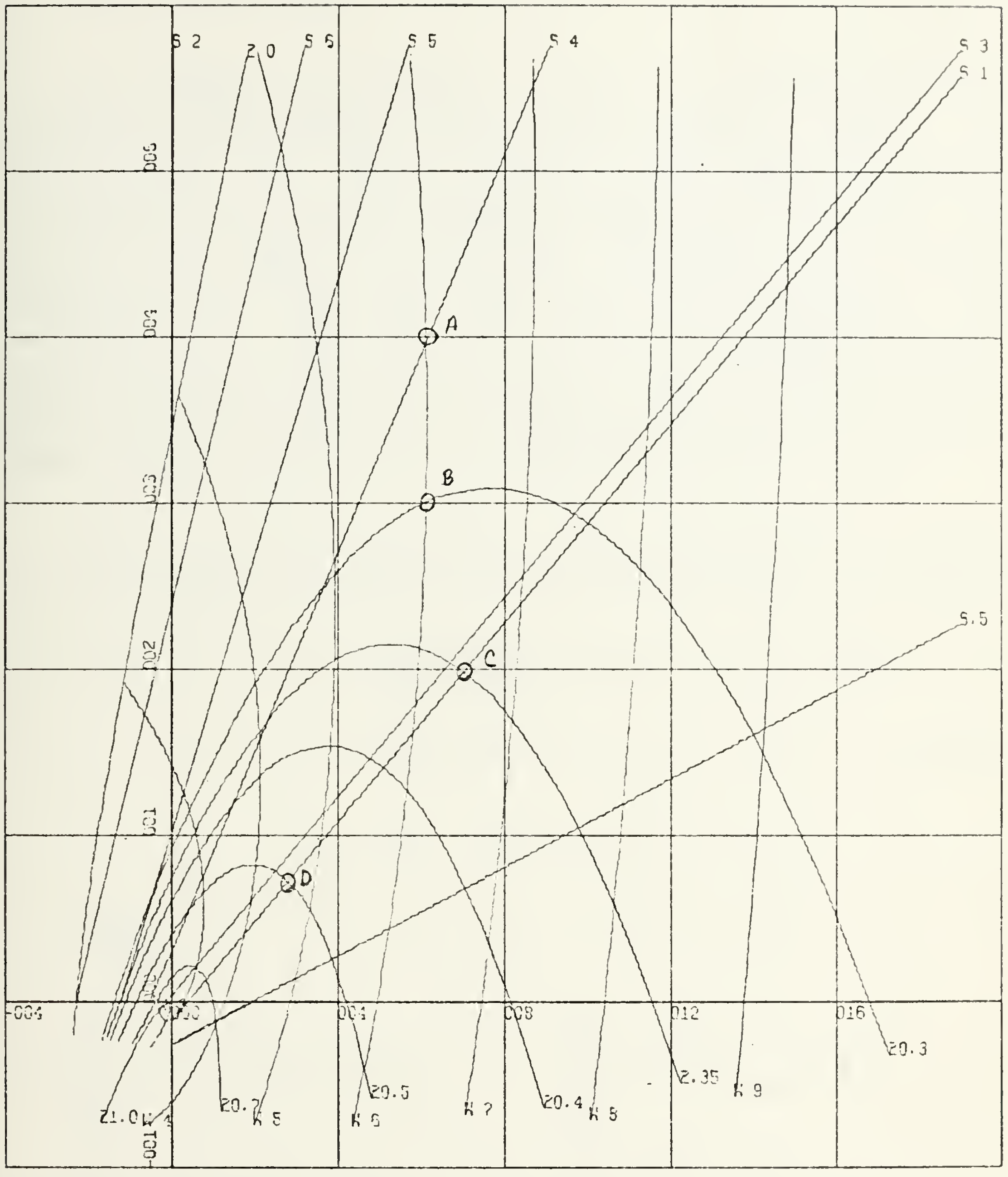
to obtain the exact roots of the system. But this is generally unnecessary as the designer will interested in the general location of a specific pair of roots.

Four points have been selected from figure 2.2. the position of these points was measured with an engineer's scale and the resulting value of and were substituted in equation (2-25). The roots of the equation were then solved by the use of a polynomial rootfinder program on the IBM 360/67. Table II-1 shows the coordinates of the points, the resulting roots and the corresponding values of zeta, omega, and sigma. As can be seen the parameter plane provides a simple means of determining a root location while variable parameters are adjusted.

TABLE II-1

POINT	ALPHA	BETA	COMPLEX ROOTS	REAL ROOTS	OMEGA	ZETA
A	6.1	4.0	-1.461±j5.814	-1.557 -4.020	5.995	.244
B	6.1	3.0	-1.815±j5.783	-1.076 -3.794	6.061	.299
C	7.0	1.975	-2.278±j6.075	-0.998 -2.945	6.490	.351
D	2.8	0.725	-2.308±j3.982	-1.004 -2.881	4.603	.501

POINTS FROM FIGURE 2.2



α-scale: 4 units/inch
β-scale: 1 unit/inch

Figure 2.2

PARAMETER PLANE EXAMPLE
FOURTH ORDER POLYNOMIAL

B. THE SINGULAR LINE

As stated in Chapter I Bowie [4] discovered the existance of the singular line in the parameter plane. In investagating the cause of singular lines Bowie found that all previous work in the parameter plane was correct for as far as it went. It did not, however, give a full explanation of the case when both the numerator and denominator of equation (2-20) formed singular matrices. Siljak [3] did address the fact that this condition could exist, but the idea of a singular line in the parameter plane had not, up to that time, been conceived.

Bowie found that in order for singular lines to exist in the parameter plane the following conditions must be met:

$$B_1 C_2 - B_2 C_1 = 0$$

$$C_1 D_2 - C_2 D_1 = 0 \qquad (2\text{-}26)$$

$$B_2 D_1 - B_1 D_2 = 0$$

where:

$$B_1 = \sum_{k=0}^{n} (-1)^k b_k {}^{k}_{ns} U_{k-1}(\zeta_s) \qquad B_2 = \sum_{k=0}^{n} (-1)^k b_k {}^{k}_{ns} U_k(\zeta_s)$$

$$C_1 = \sum_{k=0}^{n} (-1)^k c_k {}^{k}_{ns} U_{k-1}(\zeta_s) \qquad C_2 = \sum_{k=0}^{n} (-1)^k c_k {}^{k}_{ns} U_k(\zeta_s) \qquad (2\text{-}27)$$

$$D_1 = \sum_{k=0}^{n} (-1)^k d_k {}^{k}_{ns} U_{k-1}(\zeta_s) \qquad D_2 = \sum_{k=0}^{n} (-1)^k d_k {}^{k}_{ns} U_k(\zeta_s)$$

ind: ω_{ns} and ζ_s are the constantω-constantζ_s values.

equations (2-18) are solved for in terms of β the lt is two linearly independent equations:

$$\beta = -\frac{D_1}{C_1} - \frac{B_1}{C_1}\alpha = -\frac{D_2}{C_2} - \frac{B_2}{C_2}\alpha \qquad (2\text{-}28)$$

Equation (2-28) is the singular line solution. It s a straight line in the parameter plane with a slope of C$_1$ and it intercepts the -axis at $-D_1/C_1$. All points on line represent a singular value of zeta and a singular e of omega and therefore map into a single pair of lex points in the s-plane where:

$$s = -\zeta_s\omega_{ns} \pm j\omega_{ns}\sqrt{1-\zeta_s^2} \qquad (2\text{-}29)$$

It is obvious that singular lines do not exist for all acteristic equations and when they do exist they will st only for the $_s$-ω_{ns} pairs which will satisfy equation 26). To aid in determining the existance of singular es and plotting them in the parameter plane the computer gram SINGULAR LINE has been written.

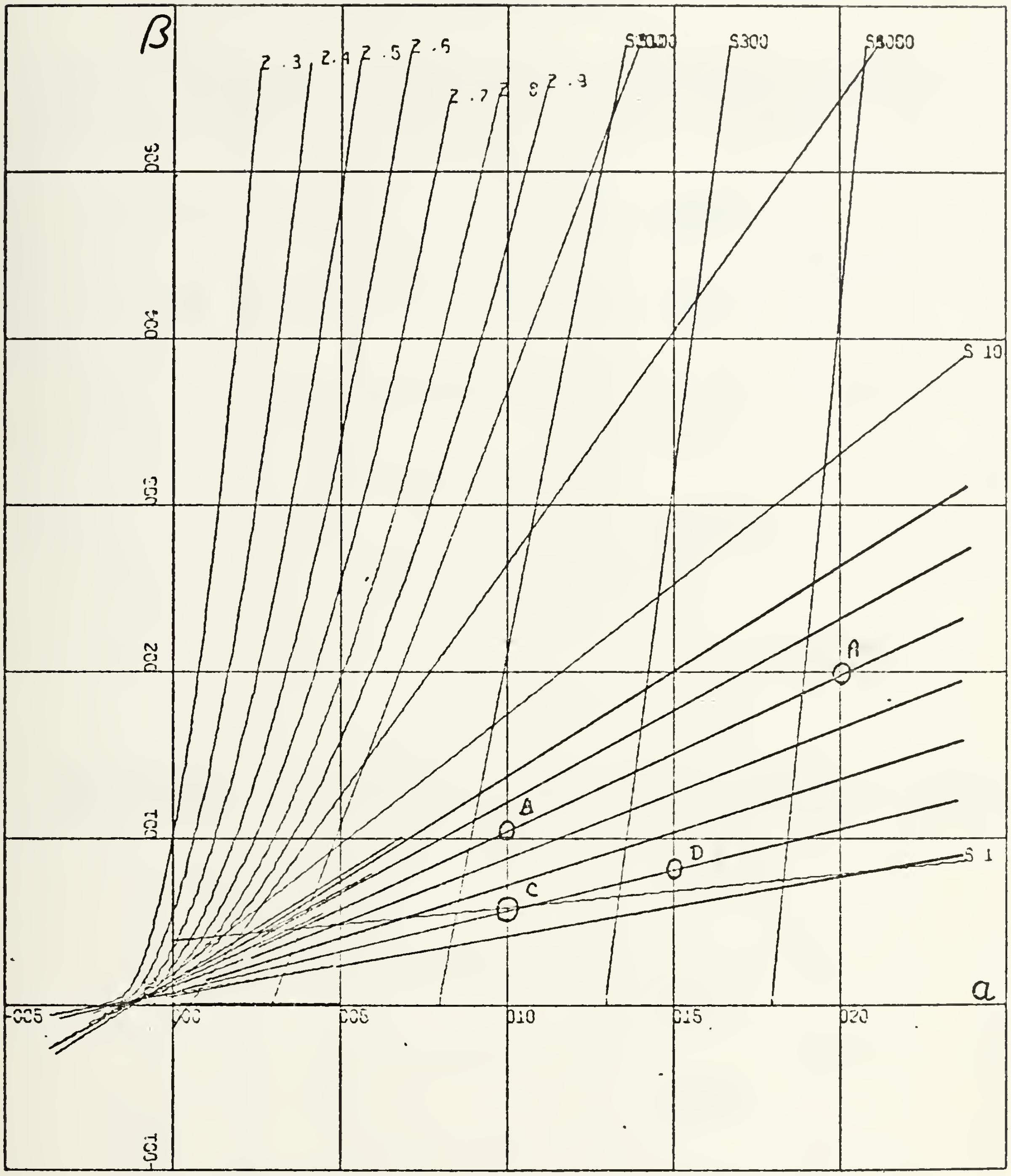
Briefly, the program uses the real constants of the fficients of the characteristic equation and the desired ue of ζ_s to solve for value for any real positive values ω_{ns} in equation (2-26). The ζ_s and ω_{ns} values are then d to generate equations (2-27) and these results are used test for the existance of singular lines. If singular

region there will be two real roots and a pair of complex roots corresponding to the constant zeta-constant omega values of the singular lines.

A pair of points has been selected along two of the singular lines. Points A and B are on the line for ζ=0.7, ω_n=13.28 and point C and D are on the line for ζ=0.4 ω_n=12.309. Table II-2 shows the coordinates, the resulting roots and the values of zeta and omega. For Table II-2 the coordinates of the point were measured with an engineer's scale and the values of alpha and beta were substituted into equation (2-30) and the roots were computed using a polynomial rootfinder subroutine on the IBM 360/67.

Table II-3 is for the same points, the only difference being that the coordinates were computed using the slope-intercept method utilizing the printed output from the program SINGULAR LINE.

Table II-3 shows that this method of obtaining system roots provides an accurate method for obtaining singular lines and that the lines a for constant zeta-constant omega values, while Table II-2 shows that reasonably accurate results can be obtained using a minimum of equipment.



α-scale: 5 units/inch
β-scale: 1 unit/inch

Figure 2.3

SINGULAR LINES ON THE PARAMETER PLANE
FOURTH ORDER POLYNOMIAL

TABLE II-2

POINT	ALPHA	BETA	COMPLEX ROOTS	REAL ROOTS	OMEGA	ZETA
A	20.	19.67	-9.244±j10.87	-.9326 -422.6	13.40	.689
B	10.	10.5	-9.281±j9.670	-1.051 -222.4	13.40	.693
C	10.	5.667	-4.839±j11.19	-.9971 -231.3	12.20	.397
D	15.	8.167	-4.918±j11.27	-1.045 -326.7	12.30	.400

POINTS FROM FIGURE 2.3

TABLE II-3

POINT	ALPHA	BETA	COMPLEX ROOTS	REAL ROOTS	OMEGA	ZETA
A	20.	19.86	-9.298±j9.486	-1.071 -422.3	13.40	.700
B	10.	10.52	-9.298±j9.486	-1.051 -222.4	13.40	.700
C	10.	5.755	-4.920±j11.27	-.9982 -231.2	12.30	.400
D	15.	8.225	-4.920±j11.27	-1.011 -331.2	12.30	.400

POINTS FROM FIGURE 2.3

III. PROPERTIES OF THE SINGULAR LINE

To fully understand the relationship of singular lines to the parameter plane it is necessary to examine what the singular line is and what it represents.

The singular line exists as a unique situation that occurs while mapping the s-plane into the parameter plane. The necessary condition that must exist is that the determinants of the matrices which form equation (2-20) must be singular. When this occurs, a discontinuity appears in the mapping from the s-plane to the parameter plane. The result of this discontiniuty is the singular line.

The discontinuity occurs for the singular value of zeta and the singular value of omega, and it maps into the parameter plane as a straight line from plus infinity to minus infinity. The equation of this line is

$$\beta = -\frac{D_1}{C_1} - \frac{B_1}{C_1}\alpha = -\frac{D_2}{C_2} - \frac{B_2}{C_2}\alpha \qquad (3-1)$$

The slope of the line is $-B_1/C_1$ and the beta intercept is $-D_1/C_1$.

Until this writing it was thought that is a value of alpha and beta from any point along the singular line was placed in the characteristic equation, the system would have a pair of complex roots corresponding the the value of the constant zeta-constant omega of the singular line. however it is shown in the following examples that this result does

not always occur.

EXAMPLE III

Consider the following aircraft pitch stabilization system [6].

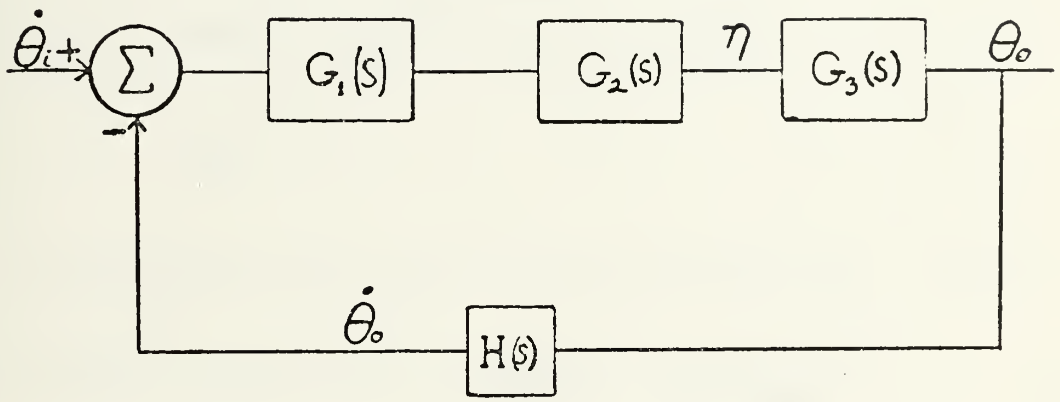


Figure 3.1
AIRCRAFT PITCH STABILIZATION SYSTEM

where: $\dot{\theta}_i$ is the pitch rate command

θ_o is the pitch rate

θ_o is the pitch angle

η is the elevator angle

and: $G_1(s)$ is a pole-zero compensator

$G_2(s)$ is a hydraulic servo

$G_3(3)$ is the airframe

H(s) is a rate gyro

The transfer functions of the system components are:

$$G_1(s) = \frac{s+\alpha}{s+\beta}$$

$$G_2(s) = \frac{K_1 3180}{s^2 + 54.85 + 3201.01}$$

$$G_3(s) = \frac{K_2 (19.41)(s + 3.22)}{s(s^2 + 7.58s + 124.614)}$$

$$H(s) = s$$

The characteristic equation for the uncompensated system is:

$$s^5 + 62.38s^4 + 3741.01s^3 + (31092.5 + 61732.6K)s^2 + (398891 + 198779K)s = 0 \qquad (3\text{-}2)$$

where $K = K_1 K_2$

This system is unstable for K>3.27. It is desired to compensate for K = 5 and to avoid the natural frequencies of the airframe, $\omega_n = 3.24$, and the hydraulic servo, $\omega_n = 7.049$

The characteristic equation of the compensated system is:

$$s^6 + (62.38 + \beta)s^5 + (3741.01 + + 62.38\beta)s^4 + (31092.5 + 61732.8K + 3741.01\beta)s^3 + (398891 + 198751K\alpha + 31092.5\beta)s^2 + (198751K\alpha + 398891\beta)s = 0 \qquad (3\text{-}3)$$

This characteristic equation with K = 5 was used with the program PARAM A to obtain the parameter plane curves, figure 3.2, and with the program SINGULAR LINE to determine

the existance of singular line. Two singular lines were found for each zeta value of 0.1, 0.2, 0.3, 0.4, and 0.5, and one singular line was found for each zeta value of 0.6, 0.7, 0.8, and 0.9.

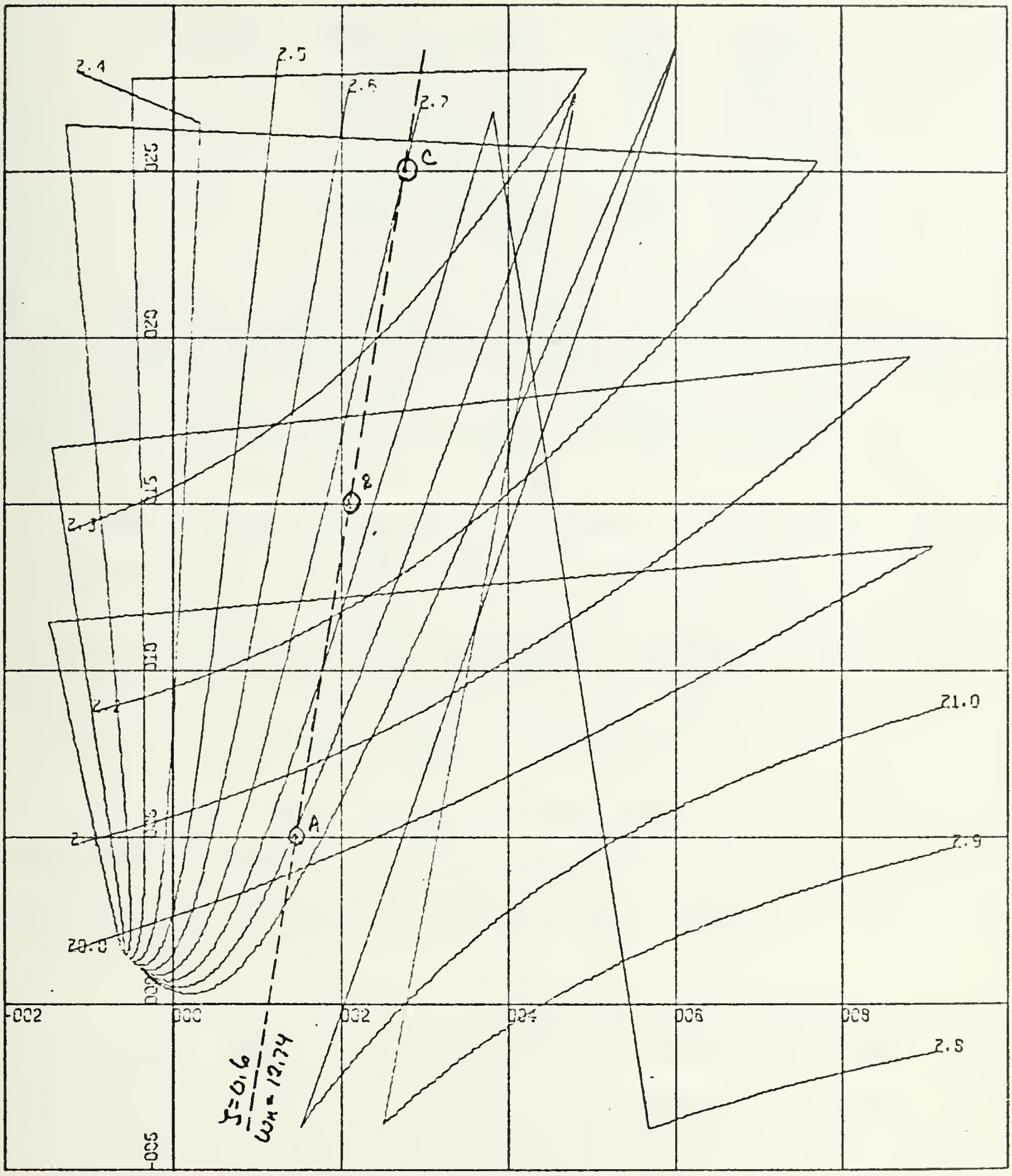
Zeta = 0.6 was chosen for compensation as it has an omega of 12.74 radians and would result in a fairly fast response without causing resonance at the natural frequencies of the airframe and the servo. The singular line for zeta = 0.6 is the broken line on the parameter plane of figure 3.2.

Three points were selected on the singular line. The points were; A(14.56,50), B(21.13,150)8 and C(27.69,250). These values of alpha and beta where put into the characteristic equation and the roots were solved for on the IBM 360 with a polynomial rootfinder routine.

There were two real roots and two pairs of complex roots for each of the points. Contrary to the supposition that all points along the singular line will produce a pair of complex roots corresponding to the value of the zeta-omega of that line, in this case such roots were <u>not</u> found. The singular line with zeta = 0.6 and omega = 12.74 should have according to prevoius concepts result in a complex root pair located at

$$s = -7.645 \pm j10.194 \qquad (3-4)$$

This was not found to be the case. The complex roots that were found did not agree with the singular line roots, but they did agree with the parameter plane roots. These results are as follows:



α-scale: 20 units/inch
β-scale: 50 units/inch

Figure 3.2

AIRCRAFT PITCH STABILIZATION SYSTEM PARAMETER PLANE WITH ZETA=0.6-OMEGA=12.74 SINGULAR LINE

POINT	COMPLEX ROOTS	ZETA	OMEGA
A	-8.083±j3.718	.908	8.897
	-2.512±j69.00	.036	69.04
B	-7.944±j6.987	.751	10.58
	-16.66±j64.41	.251	66.53
C	-7.871±j8.012	.701	11.23
	-20.75±j59.36	.330	62.87

Identical results were found when values were taken from other singular lines in the same relative area of the parameter plane. All the roots were found to agree with the parameter plane, but not with the singular lines.

The initial response to these results was that the singular lines that were found were incorrect. However, after carefully checking the program siat were found were incorrect. However, after carefully checking the program SINGULAR LINE and hand calculating two of the singular lines, the program was deemed to be correct.

In an attempt to find a solution to this dilemma the singular line examples given by Bowie were examined. In all cases the singular lines were located outside the region of the normal complex root parameter plane curves.

In the belief that the constant zeta curves in figure 3.2 formed a loop as omega increased, values of alpha and beta believed to be outside the region were tried. The straight lines connecting the zeta curves in the figures shown in this thesis are the result of the DRAW routine and are not part of the parameter plane. They result when intermediate points are not in the area of the plot. Again three points on the same singular line were selected. These points were D(76.93,1000), E(142.57,2000) and F(208.22,3000). These points are not located on figure 3.2, but the location was computed using the slope-intercept

method. These results are as follows:

POINT	COMPLEX ROOTS	ZETA	OMEGA	
D	-7.725±j9.542	.629	12.28	*
	-23.31±j51.08	.415	56.15	
E	-7.688±j9.856	.615	12.50	*
	-23.47±j49.55	.428	54.59	
F	-7.675±j9.966	.610	12.58	*
	-23.50±j49.04	.432	54.38	

It is evident from the above that one pair of complex roots, indicated by the astrisk, is approaching the value of those which should result due to the singular line and that the value of zeta and omega for that pair of roots is relatively insensitive to large changes in alpha and beta.

In light of these results an examination of the parameter plane in the region of these points was conducted. For reasons of clarity only three values of zeta were plotted. These were 0.0, 0.6, and 1.0. The resulting parameter plane is seen in figure 3.3. As can be seen the constant zeta lines do not loop around as prevoiusly thought, but are discontinuous and the singular line approaches asymptotically.

What has happened is that as omega was increased on the constant zeta curve and approched the singular point the matrix which forms the denominator of equation (2-20) becomes very much smaller than the numerator causing the constant zeta line to go to plus infinity at the singular point asymptotic to the singular line. Once the value of omega is past the singular point it reenters the parameter plane at minus infinity, asymptotic to the singular line. Since there is only one singular point for this value of zeta the constant zeta line will continue through the parameter plane approaching infinity once more as omega

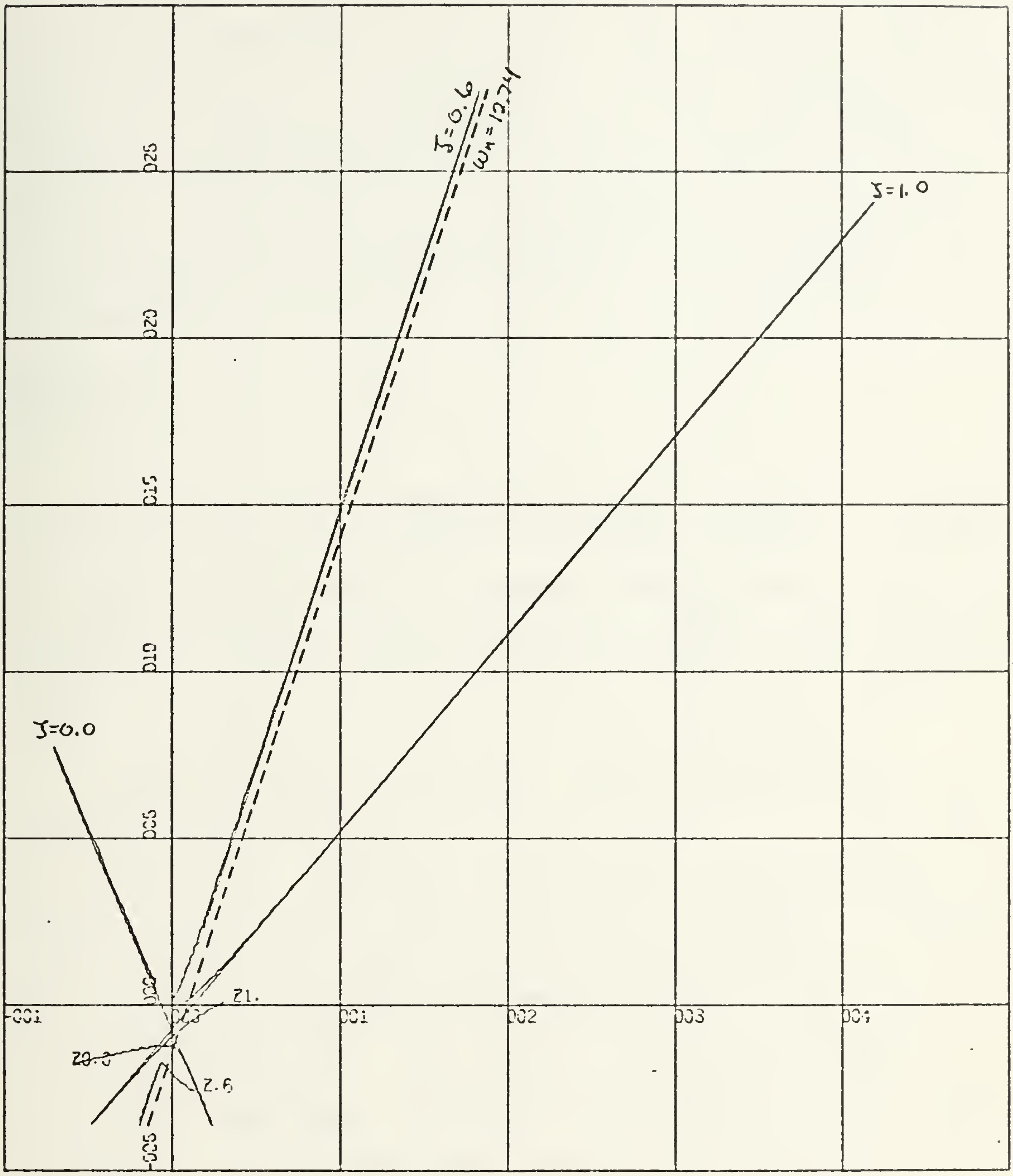
approaches infinity.

The reason that this occurs is evident when the mapping of the constant zeta line form the s-plane to the parameter plane is observed. The constant zeta line in the s-plane is continuous form $\omega_n = 0$ to $\omega_n = \infty$, therefore the mapping of this line into the parameter plane must also be continuous. However there is a discontinuity at the singular point where the constant zeta curve in the paramter plane goes to infinity for a finite value of omega. The singular line provides a continuous mapping of the constant zeta curve as it traverses from plus infinity to minus infinity. This line exists at infinity, but not in the parameter plane as it is known.

In an effort to examine the change in alpha and beta as omega was increased a printout of PARAM A was made. However due to the iteration of omega in PARAM A it was not possible to obtain sufficient data around the singular point. In PARAM A for an omega of about 12 radians the iterative step is about 0.9 radians. Therefore the section of PARAM A which computes the constant zeta curves was modified to vary omega over 1 radian in steps of 0.001 radians starting 0.5 radians below the singular point and increasing to 0.5 radians above the singular point.

In order to see how small increases in omega affect the values of alpha and beta some of the points are shown:

OMEGA	ALPHA	BETA
12.242	88.56	1297.87
12.342	111.31	1644.01
12.442	149.21	2220.85
12.542	226.06	3391.11
12.642	451.58	6826.15
12.741	144447.	2200380.
12.743	-66473.4	-1012672.
12.842	-454.96	-6980.32
12.942	-230.44	-3564.49
13.042	-154.32	-2405.44



α-scale: 100 units/inch
β-scale: 500 units/inch

Figure 3.3

PARAMETER PLANE FOR AIRCRAFT PITCH STABILIZATION SYSTEM
ZETA = 0.6

As can be seen for large changes in alpha and beta in the area of the parameter plane where the singular line is asymptotic to the constant zeta curves results in small changes of omega. Since omega is very insensitive to changes on alpha and beta in this region the concept of the singular line is valid from the standpoint of the engineer. It is possible to vary a parameter along the singular line in this region and expect to maintain a constant value of zeta and omega.

EXAMPLE IV

In light of the unexpected results obtained in example III it was decided to carefully reexamine example II that was given in the previous chapter. The parameter plane was expanded around the area of the origin where the singular lines pass through the complex root portion of the parameter plane.

Figure 3.4 shows this area of the parameter plane. The constant zeta curves are for values of 0.0, 0.2, 0.5, 0.6, 0.8, and 1.0. There are two singular lines for each zeta value of 0.2, 0.5, 0.6, and 0.8 which are shown as dashed lines. There are two discontinuities in each zeta curve. Although it is not immediately obvious due to the limited area of the plot, all the constant zeta curves originate in the second quadrant, and become infinite at the first dicontiuity and then reappear at infinity in the fourth quadrant. The curves, for values of zeta up to and including 0.5 then follow a path just to the left of the origin and enter the first quadrant where they again become discontinuous and infinite. The zeta curves for values of zeta greater than 0.5 come from infinity in the fourth quardant and go to the left of the origin where they turn into the third quadrant where they again become

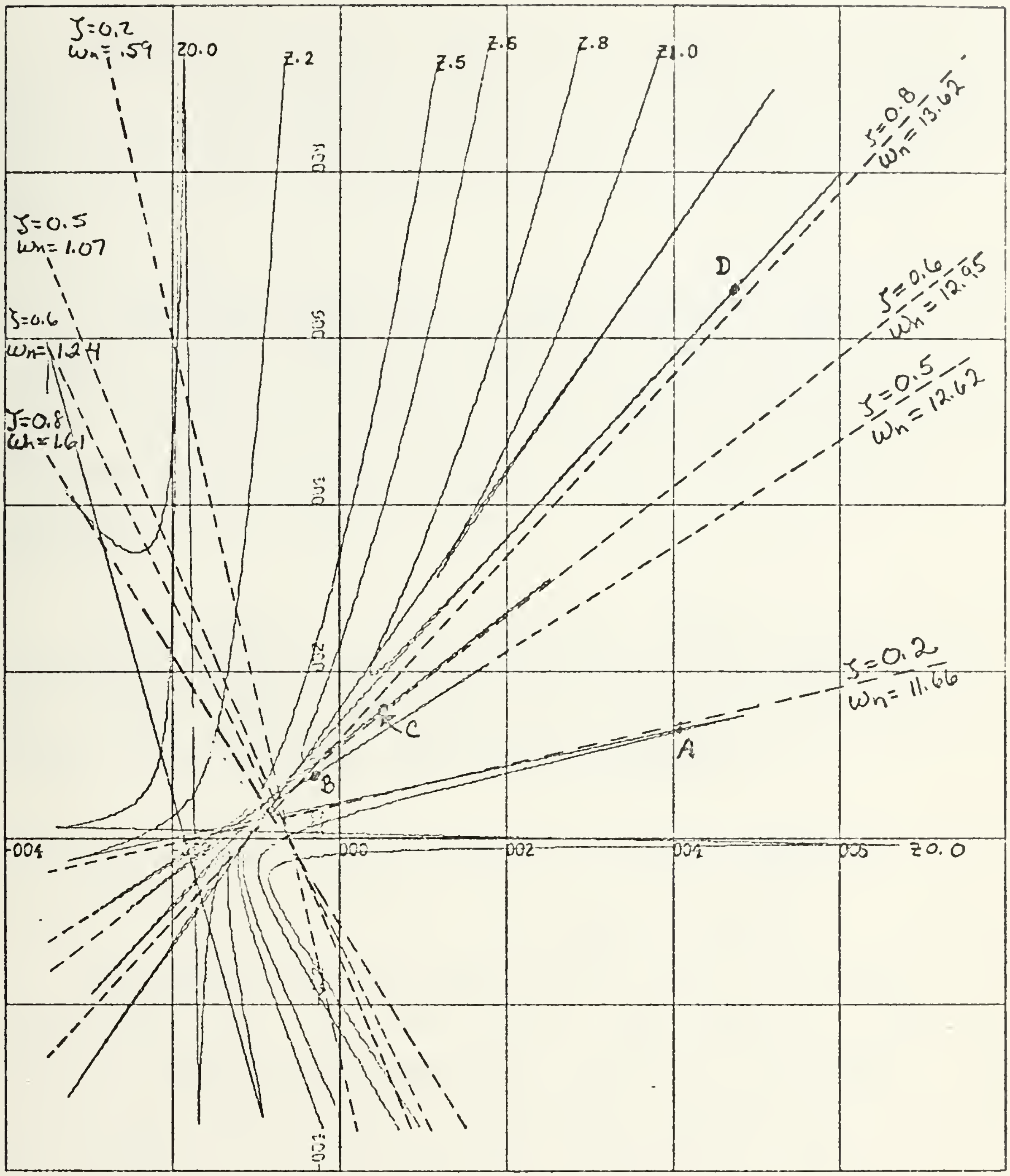
discontinuous and infinite. The curves for zeta values of 0.5 and less reappear at infinity in the third quadrant then go to infinity as a continuous function in the first quadrant. The curves for zeta values greater than 0.5 reappear at infinity in the first quadrant, sweep toward the origin then back through the first quadrant to infinity

The region of stability in the parameter plane is bounded by the zeta=0.0 curve, therefore the singular lines of interest lie in the first quadrant. As can be seen the zeta=0.5 and zeta=0.6 singular lines nearly coincide with the corresponding zeta curves in the parameter plane and the zeta=0.2 and zeta=0.8 lines are being asymptotically approached by the corresponding parameter plane zeta curves.

As in example III the modified section of PARAM A was used to abtain a printout of alpha and beta for a 1 radian change in omega as the constant zeta curve passed through the singular line value. In this example the constant zeta curves in the first quadrant approached their singular value very rapidly.

For purposes of illustration points have been placed on the constant zeta curves in figure 3.4 at the value at which omega is within 0.5 radians of the singular point which is at infinity. These points are:

POINT	ZETA	ALPHA	BETA
A	0.20	4.078	1.30
B	0.50	-0.542	0.59
C	0.60	0.571	1.60
C	0.80	4.754	6.62



-scale: 2 units/inch
-scale: 2 units/inch

Figure 3.4

SINGULAR LINE THROUGH THE PARAMETER PLANE
FOURTH ORDER POLYNOMIAL

As can be readily seen these points are within 5 percent of the singular value of omega and if one continues along the zeta curves it will be found, that for practical design purposes, the changes in omega are inconsequential.

It has been shown that a true singular line in the original concept does not exist. In examples III and IV it is shown that, if a designer carefully examines the singular lines and their relationship to the constant zeta curves of the parameter plane the singular concept can be used for practical design applications.

EXAMPLE V

This example is the original system in which Bowie found that singular lines existed in the parameter plane. The system which was used has a characteristic equation which is the following sixth order polynomial:

$$s^6 + 80s^5 + (20\alpha+1600)s^4 + 840\alpha s^3 + (1600\alpha+400\beta)s^2 + 1600\beta s + 1600\beta = 0 \qquad (3\text{-}5)$$

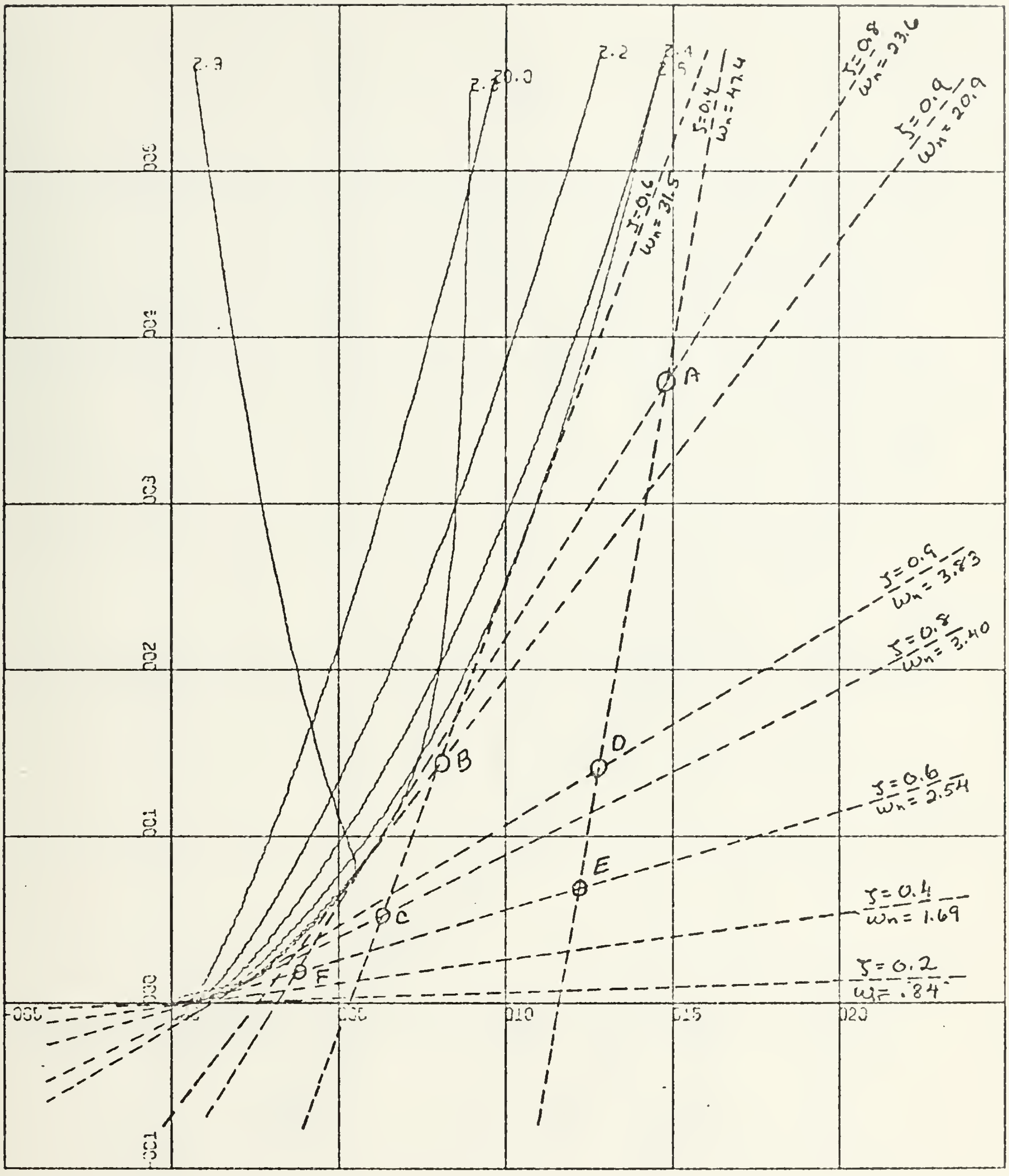
The parameter plane with constant zeta curves with the values of 0.0, 0.2, 0.4, 0.6, 0.8, and 0.9 is seen in figures 3.5 and 3.6. Figure 3.6 is the area around the origin in figure 3.5. There were two singular lines for each of the zeta values 0.2, 0.4, 0.6, 0.8, and 0.9 and these are shown on figures 3.5 and 3.6 as dashed lines.

As can be seen the parameter plane has no visible discontinuities as omega increases, however due to the fact that two singular lines exist for each value of zeta there must be two discontinuities in each zeta curve. Examination of the singular lines and the constant zeta curves of the

parameter plane indicates that the singular lines are tangent tc the corresponding zeta curve at one point. When the values of zeta and omega for these points were examined it was found that the tangent points occur at the singular points in the constant zeta curves.

Unlike examples III and IV where the mapping from the s-plane approached infinity as the singular pcint was approached this example maps as though no discontinuity existed. When the discontinuity is reached the constant zeta curve immediately goes to plus infinity tangent to the curve at the point of discontinuity and returns from minus infinity tangent to the constant zeta curve and continues mapping to infinity as omega approaches infinity.

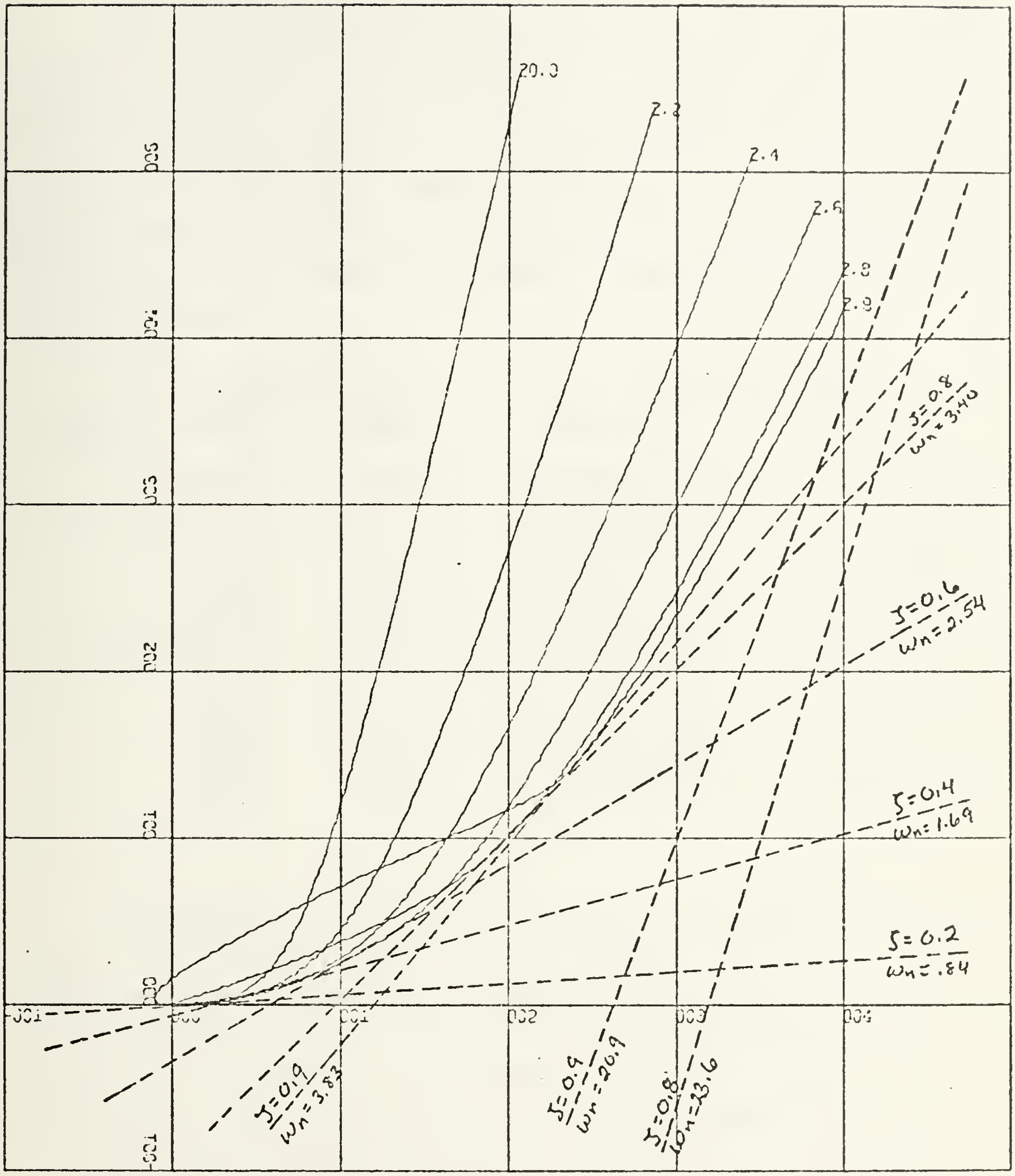
This phemonenon does not reveal itself in a normal parameter plane study for two reasons. The first of these is the previously mentioned size of iteration in the PARAM A program and the second reason is that a normal practice in computer programing is to limit the numerator of a term to a minimum size to prevent an overflow or underflow in the computer when a number beyond the computer's capablity is reached. This practice was followed in the writing of PARAM A so if the singular point was reached on an iterative step the program would have recognized this as a forbidden number and continued to the next value of omega, thereby bypassing the singular point and continuing with no indication that this condition existed. If Bowie had not been interested in the area outside the normal complex root area of this example he would have had no indication that singular lines existed.



α-scale: 50 units/inch
β-scale: 1000 units/inch

Figure 3.5

SINGULAR LINE IN THE PARAMETER PLANE
EXAMPLE V



α-scale: 10 units/inch
β-scale: 100 units/inch

Figure 3.6

SINGULAR LINES IN THE PARAMETER PLANE
EXAMPLE V - EXPANDED

In this example the singular lines are true singular lines as originally conceived. If a value of alpha and beta is selected at any point along these lines a pair of complex roots corresponding to the constant zeta-constant omega of that line will result. To show this fact six points have been selected in the region of singular line on figure 3.5. As can be seen by the following the expected complex roots were obtained.

POINT	SINGULAR ZETA	POINT OMEGA	COMPLEX ROOTS EXPECTED	OBTAINED
A	0.80 0.40	23.55 47.41	-18.44±j14.13 -18.96±j43.15	-18.42±j14.15 -18.95±j43.44
B	0.90 0.60	20.88 31.53	-18.79±j9.101 -18.92±j25.22	-18.79±j9.121 -18.91±j25.23
C	0.80 0.60	3.396 31.53	-2.717±j2.078 -18.92±j25.22	-2.721±j2.081 -18.91±j25.22
D	0.90 0.60	3.831 31.53	-3.450±j0.728 -18.92±j25.22	-3.465±j0.729 -18.90±j25.23
E	0.60 0.40	2.537 47.41	-1.522±j2.030 -18.96±j43.45	-1.524±j2.032 -18.94±j43.43
F	0.60 0.80	2.537 23.55	-1.522±j2.030 -18.44±j14.13	-1.521±j2.029 -18.47±j14.15

In summary, the singular lines result when a constant zeta or constant omega curve is mapped from the s-plane to the parameter plane and a discontinuity occurs. At this discontinuity the parameter plane mapping goes to infinity, either plus or minus, and returns from the other side of infinity. The singular line simply provides a mapping through this region to maintain a continuous mapping from the s-plane to the parameter plane. In cases, such as examples III and IV, where the mapping first approaches infintiy as the singular point is approached the singular line concept is a valid engineering aid over a limited range of the singular line. However in cases such as example V the singular line may be used over its entire range as an

excellent design aid.

IV. THE SINGULAR SURFACE IN THE PARAMETER SPACE

A. PARAMETER SPACE

The parameter space is an extension of the parameter plane when there are three or more variable parameters [7], [8], [9]. Once the concept of the parameter plane is understood it is relatively easy to extend the concept to three or more variable parameters.

The easiest method to generate a three dimensional parameter space is to set the third variable parmeter the third variable parameter to a known value and then generate the two dimensional parameter plane. The process is then repeated for additional values of the third parameter. The result is a series of parameter planes which will form a parameter space.

EXAMPLE VI

Consider the following system:

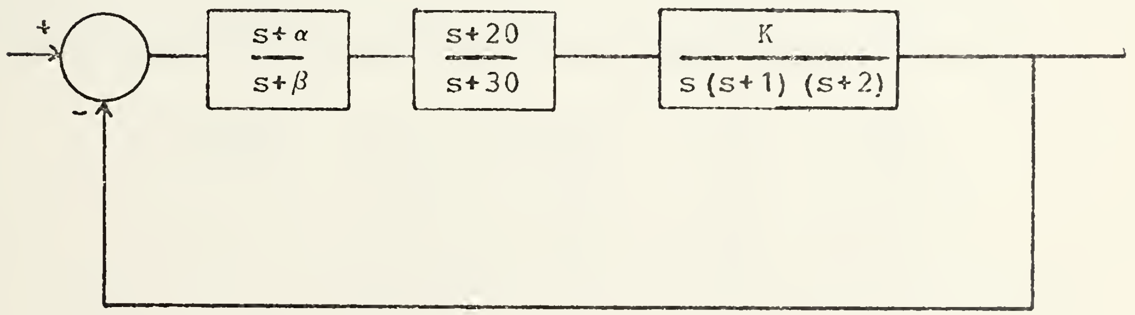


Figure 4.1
THIRD ORDER SYSTEM WITH DUAL CASCADE COMPENSATION

The characteristic equation for this system is:

$$s^5 + (33 + \beta)s^4 + (92 + 32\beta)s^3 + (60 + 92\beta + K)s^2 + (20K + \alpha K + 60)s + K\alpha = 0 \qquad (4\text{-}1)$$

Figure 4.2 shows the parameter space for the constant zeta=0.5 and constant omega=1.7 and the values of K from 10 to 15. It was constructed by fixing the value of K and computing the parameter plane using PARAM A. The curves were then plotted onto a single plot to show the three dimensions.

Both the constant zeta curves and the constant omega curves form a surface and the interconnection of these surfaces forms a curved line through the parameter space. Any point on this line will generate a pair of complex roots corresponding to the s-plane point of zeta=0.5 and omega=1.7. For these values of zeta and omega the complex roots are:

$$s = -0.850 \pm j1.470 \qquad (4\text{-}2)$$

By selecting three points on this line this is shown to be true. The points are A(0.16, 2.6, 10.0), B(0.347, 3.0, 12.0), and C(0.52, 3.63, 15.0). These points were taken from the parameter plane using an engineer's scale.

POINT	COMPLEX ROOTS	ZETA	OMEGA
A	$-0.847 \pm j1.478$	.497	1.703
B	$-0.849 \pm j1.475$	.498	1.701
C	$-0.851 \pm j1.469$	.501	1.698

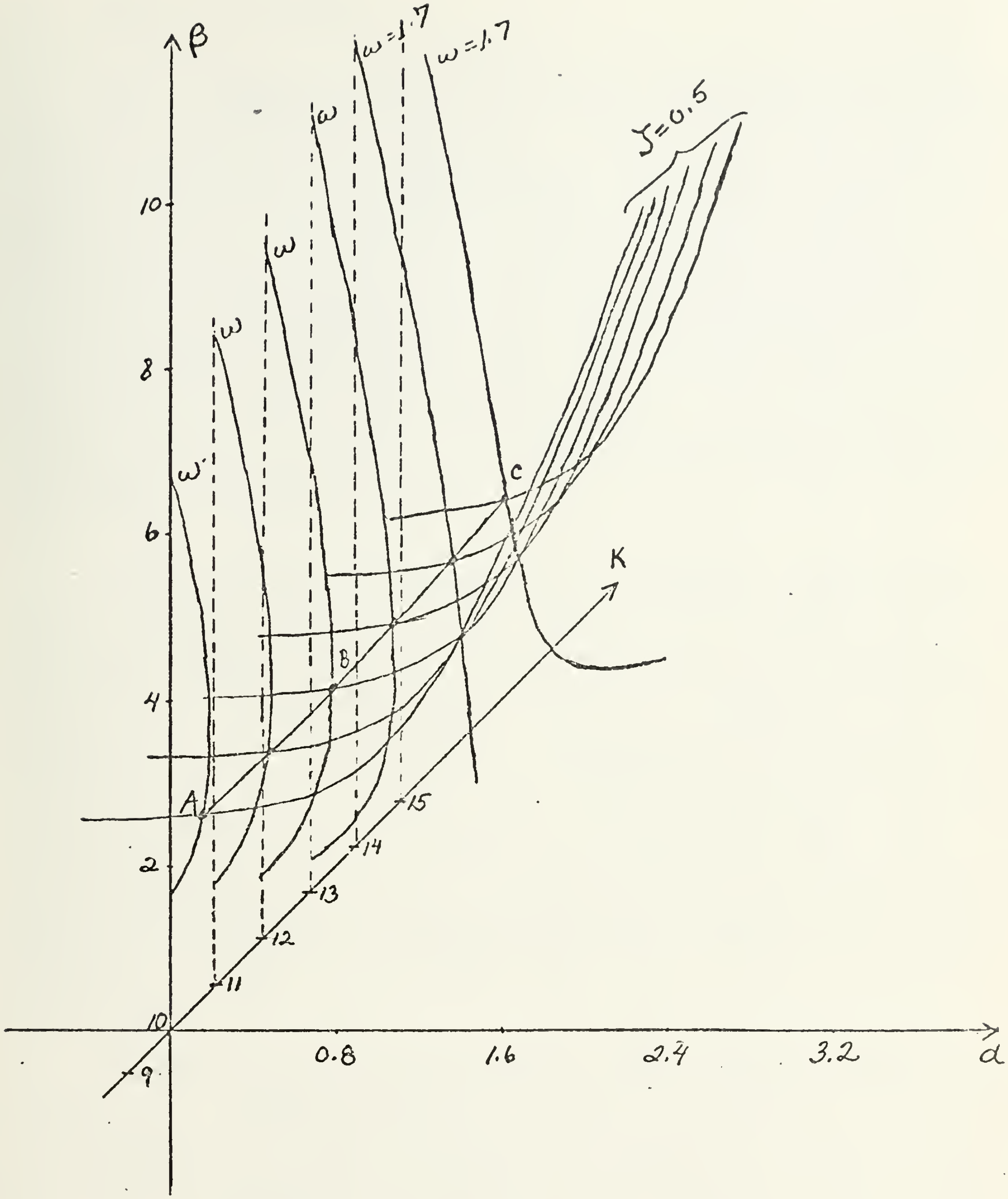


Figure 4.2

PARAMETER SPACE WITH ZETA=0.5 AND OMEGA=1.7 SURFACES

B. SINGULAR SURFACES

The parameter space is continuous throughout the range of the third variable parameter. If for any value of the third parameter a discontinuity appears in the parameter plane it will also appear in the parameter space as a discontinuity. As stated in chapter III this discontinuity will result in a singular line. As the third parameter is varied the discontinuity will result in singular lines for the same value of zeta and omega. The slope and the axis intercept points will change slighty so the result is a smooth surface representing a warped hypersurface traversing through the parameter space.

Figure 4.3 is a photograph of a singular surface model. The singular surface is from the system in example VI. the singular values are zeta=0.5 and omega=12.04 and the third variable K is varied from 5.0 to 20.0. As can be seen the surface is a smooth warped hyperplane. The model was constructed by obtaining the singular lines for the same singular value at each value of K, transferring them to cardboard, cutting them and accentting them with a contrasting color. The photography was done by the Photo Division, Education Media Department, Naval Postgraduate School.

Figure 4.4 is a drawing of the same singular surface as figure 4.3. The difference between the drawing and the photograph is that in the photograph the K axis starts at K=5 in the foreground continues back through to K=20, while in the drawing K=5 is the most distant singular line and the singular lines progress forward until the value of K=20 is reached.

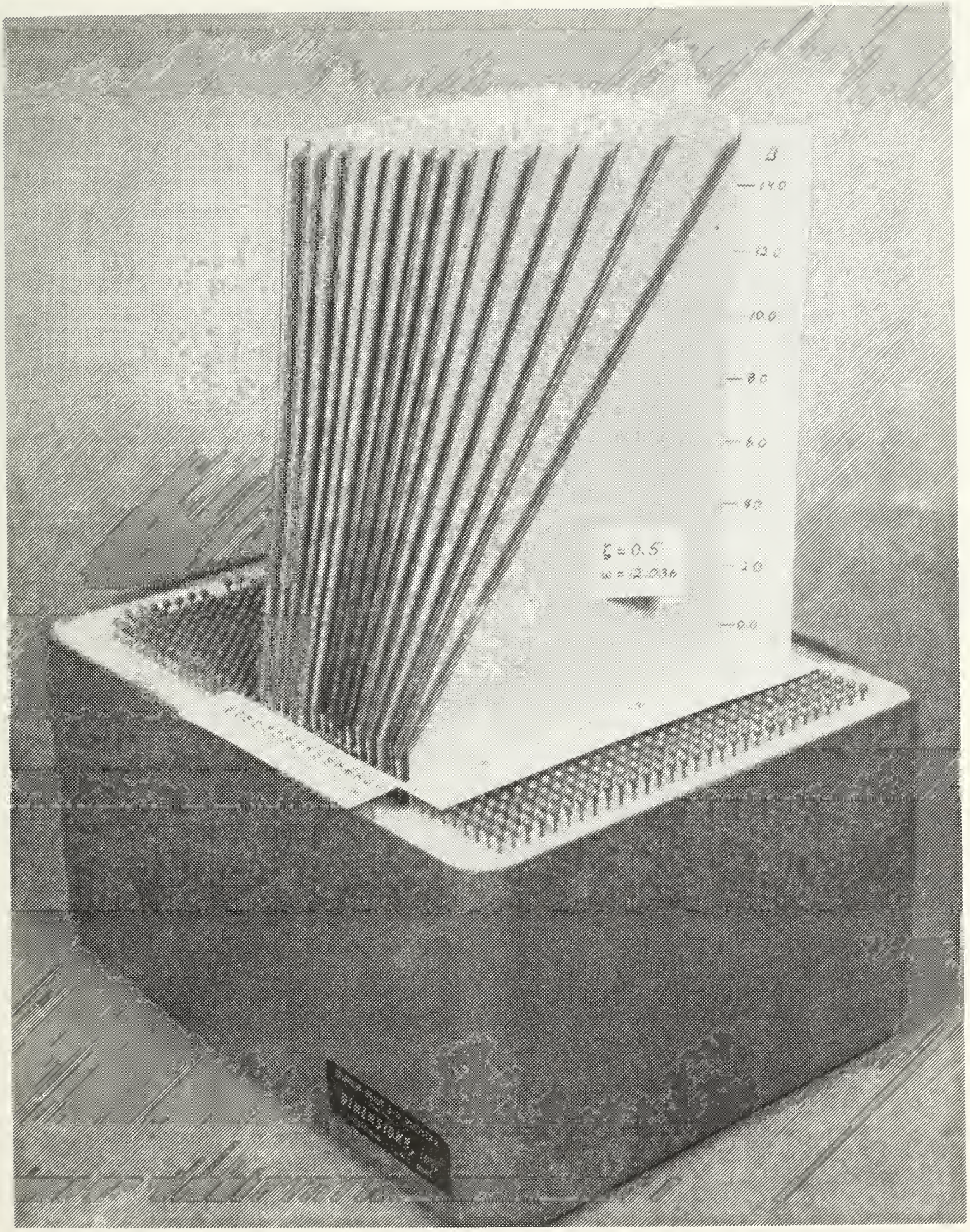


Figure 4.3
SINGULAR SURFACE MODEL

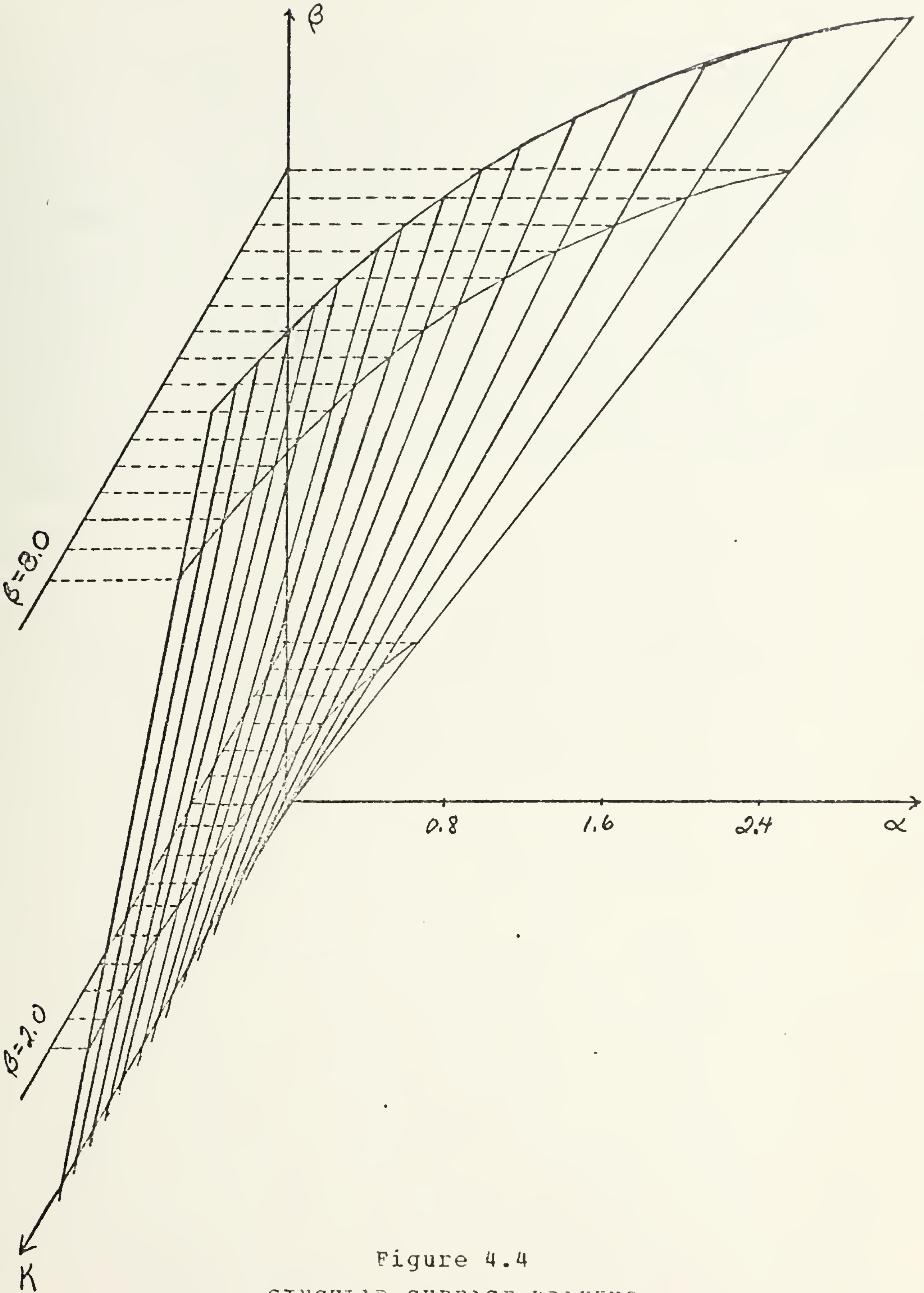


Figure 4.4
SINGULAR SURFACE DRAWING

The relationship of the singular surface to the parameter space is exactly the same as that of the singular line to the parameter plane. As with the singular line, the singular surface provides continuity at infinity. The designer must be aware of the type of discontinuity which is producing the surface. If the discontinuities occur like the ones in example III and IV then the principles of singular lines may only be applied in limited areas of the surface. If the singularities occur like those in example IV then any point on the singular surface will give a pair of complex roots corresponding to the constant zeta-constant omega of the surface.

V. COMPENSATION TECHNIQUES USING THE SINGULAR LINE

The major advantage of using singular lines for compensation is that as one moves up and down the singular line, within previously mentioned limits, one pair of complex roots remains in the same locations in the s-plane. While this pair of roots is stationary other root locations can be varied as the location on the singular line is moved through other singular lines, real root lines, and other constant zeta parameter plane curves.

There many excellent texts available which go into great detail on how root location will affect system response, overshoot, damping and so forth [1], [10]. Once the basic principles on how root locations affect a system are understood it is possible to use singular line concepts to obtain proper root locations.

EXAMPLE VIII

Consider the pitch stabilization system given in example III.

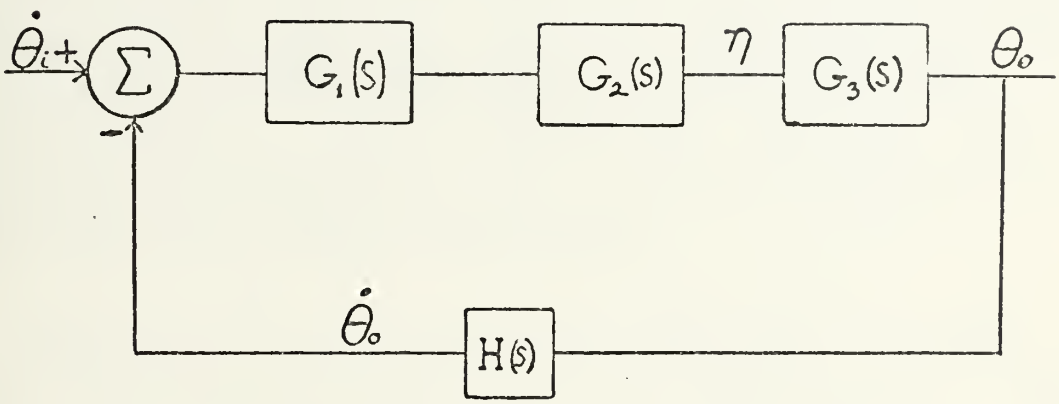


Figure 5.1
AIRCRAFT PITCH STABILIZATION SYSTEM

The characteristic equation for the system is:

$$s^6 + (62.38 + \beta)s^5 + (3741.01+ + 62.38\beta)s^4$$
$$+ (31092.5 + 61732.8K + 3741.01\beta)s^3 + (398891 + 198751K\alpha$$
$$+ 31092.5\beta)s^2 + (198751K\alpha + 398891\beta)s = 0 \qquad (5-1)$$

It was shown in example III that singular lines exist for this system, but they are the result of discontinuities at infinity. Therefore in order to effectively use the singular line concept it is necessary to examine the singular lines to determine the areas in which singular results could be expected.

In the case of this problem it was decided to use the singular line with zeta=0.6 and omega=12.74 as a dominant

root pair to give fast system response and avoid the natural frequencies of the servo and the airframe. Figure 5.2 shows the singular lines for this system for values of zeta of 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.

By moving up the zeta=0.6, omega=12.74 singular line above the region where beta=3000 the value of zeta and omega should remain relatively constant. Two points have been selected to attempt compensation, A(273.87, 4000) and B(306.69, 4500). These values have been put into the characteristic equation and the system was simulated using a step input of 0.2 radians per second.

Figure 5.3 is the response for the point A values of alpha and beta and figure 5.4 is the response for the point B values of alpha and beta. As can be seen the system response is essentially the same for both points. The response from the point A values has a peak overshoot of 1.08 and the response from the point B values has a peak overshoot of 1.1. The steady state output is 1.66 degrees in both cases.

This demonstrates one of the main advantages of the singular line concept. As two parameters are varied the system response remains insensitive to the change.

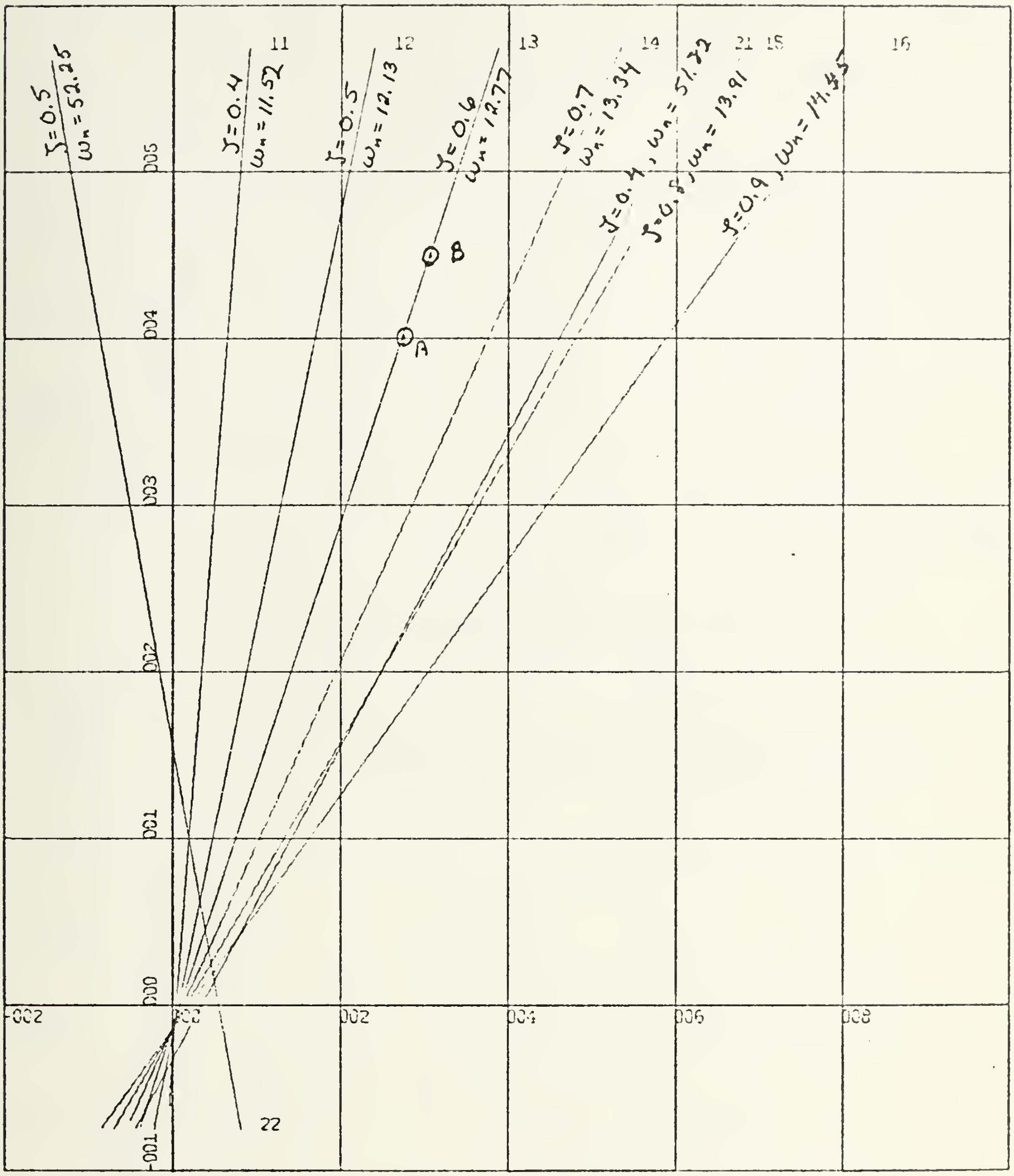
If this parameter variation were to take place inside the normal complex root region of the parameter plane the shift of root locations would result in a change of output response. Figure 5.5 is the parameter plane of the normal complex root area of the airdraft pitch stabilization system. Between point A and point C the linear change in beta is 200 and the change in alpha is 13.13.

There are two pairs of complex roots and two real roots in this section of the parameter plane. One of the real

roots is cancelled at the origin of the s-plane by the zero produced by the rate gyro in the feedback loop. The other real root location is past s=-75 in these cases and will only have a slight affect on the steady state value of the output. The complex root locations and their corresponding values of zeta and omega are:

POINT	COMPLEX ROOTS	ZETA	OMEGA
A	-8.083±j3.718	.908	8.897
	-2.512±j69.00	.036	69.04
B	-7.944±j6.987	.751	10.58
	-16.66±j64.41	.251	66.53
C	-7.871±j8.012	.701	11.23
	-20.75±j59.36	.330	62.87

At point A the dominant root is located at s = -2.512 ± j69.00. At point C the dominant root is located at s = -7.871 ± j8.012. It can be seen that at the point A values the system response will be almost undamped with a peak overshoot of about 1.9 and a natural frequency of about 69 radians, while at the point C values the system will be well damped with a natural frequency of about 11 radians.



α-scale: 200 units/inch
β-scale: 1000 units/inch

Figure 5.2

SINGULAR LINES FOR
AIRCRAFT PITCH STABILIZATION SYSTEM

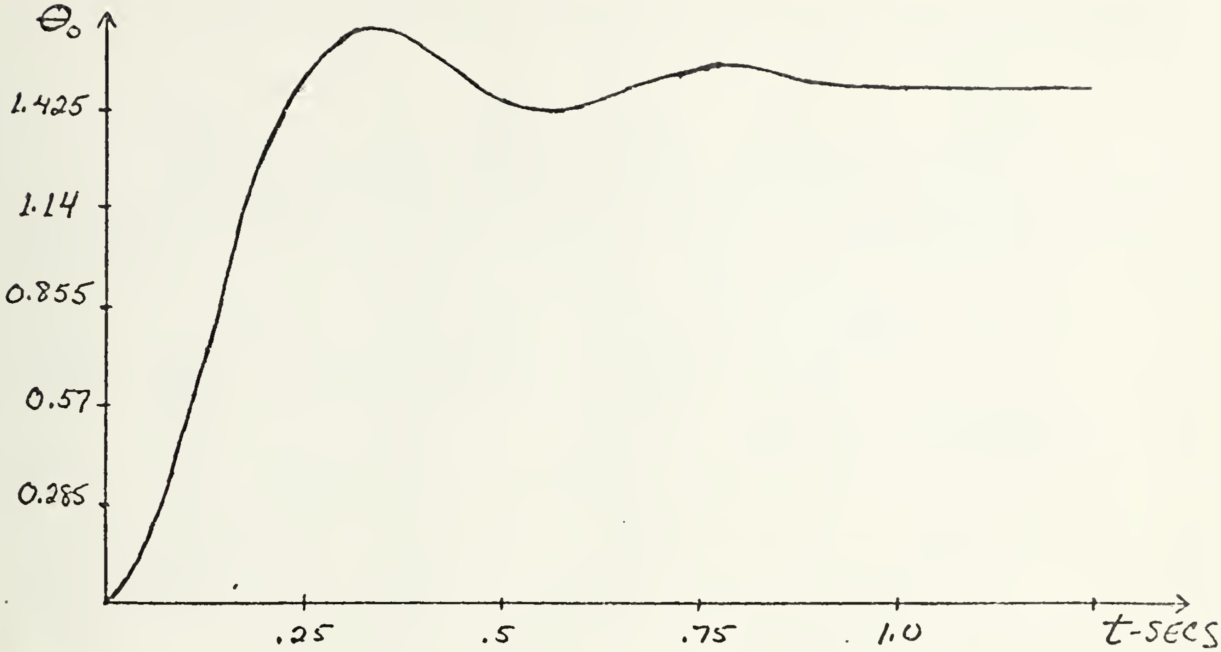


Figure 5.3

STEP RESPONSE FOR POINT A

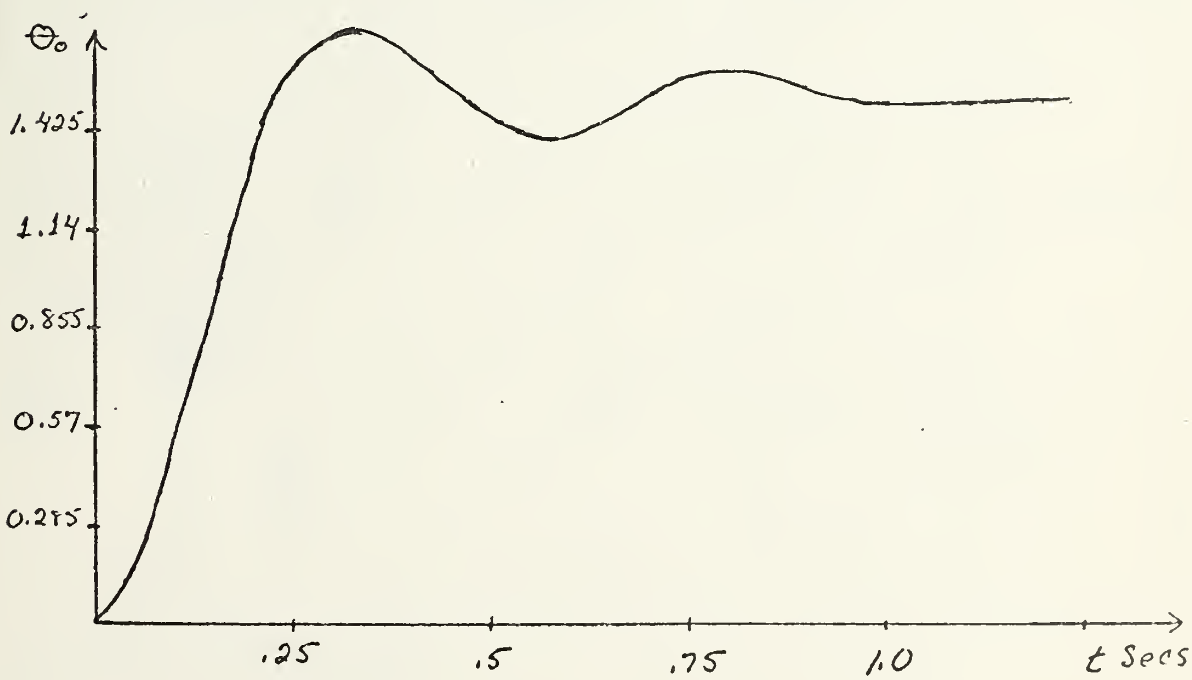
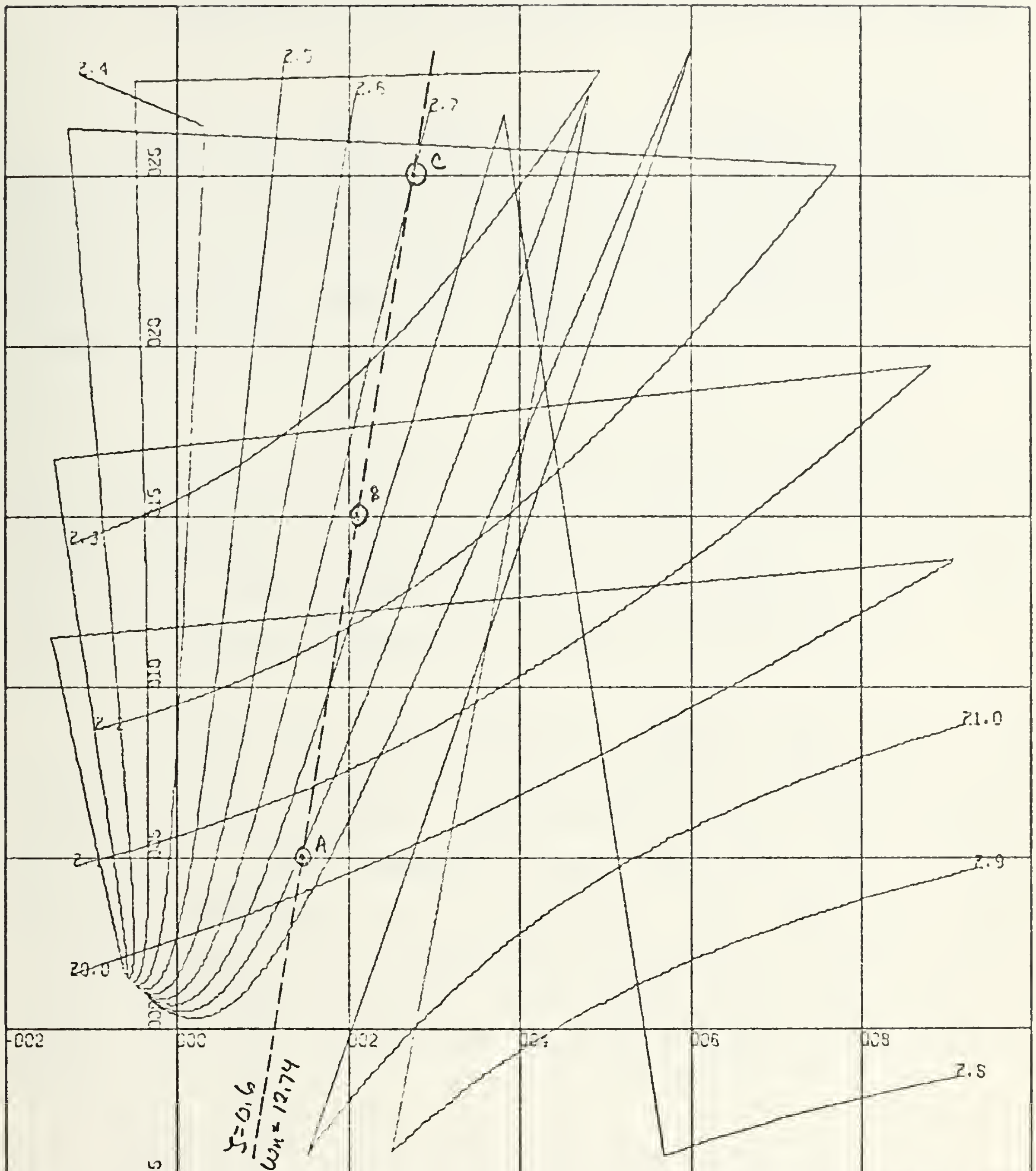


Figure 5.4

STEP RESPONSE FOR POINT B



α-scale: 20 units/inch
β-scale: 50 units/inch

Figure 5.5

AIRCRAFT PITCH STABILIZATION SYSTEM PARAMETER PLANE

VI. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

It has been shown that some of the original concepts of the singular line were in error due to previously unknown properties of the singular line and its relationship to the parameter plane. The main reason these properties were not found earlier was due tmainly to standard computer programming practices and limitations.

Although it has been found that some limitations exist when using singular line concepts it has been shown that the basic concepts of singular lines can be used by the design engineer to easily obtain desired system response when more than one variable parameter is present in the system.

An example has been presented which shows that the singular line concept is an excellent method to obtain desired system response. The method will allow the designer to select either dominant root locations or, if it is desired, nondominant root locations. the root locations when selected from the singular line region of the parameter plane will remain fixed in their location as long as the alpha and beta parameters are restricted to the singular line.

For further study it is highly recommended that the case where true singular lines exist such as in example V be thoughly studied to obtain a full understanding of why the singular point does not occur at infinity in the parameter plane. Once this is accomplished devise a method to force this type of condition at a desired zeta-omega value. If this can be accomplished it would allow the designer to

select a root location in the s-plane and use singular line concepts for easy design and compensation.

APPENDIX A

PARAM A

This is a modified version of the PARAM A program written by Nutting [5]. Originally written in FORTRAN for the CDC 1604 Computer, it has been modified to FORTRAN IV for the IBM 360/67 at Naval Postgraduate School. It has also been modified to allow the user to select the size of the output plot and to plot singular lines in the parameter plane.

However PARAM A will not determine the existance of singular lines. It is necessary for the user to know the value of ζ and ω_n of a singular line before this option can be used. These values can best be determined by use of the program SINGULAR LINE. The user then can either put these values into PARAM A or use the values to draw the singular line on the PARAM A output plot.

APPENDIX B

SINGULAR LINE

The purpose of the program SINGULAR LINE is to determine the existance of singular lines in the parameter plane. The program is written in FORTRAN IV for the IBM 360/67 computer and the CALCOMP plotter at Naval Postgrauate School.

The values read into the program are the coefficients of the characteristic equation, the values of zeta to be examined, and plotting information. The program contains three subroutines which are located in the source library of the IBM 360/67. These are POLRT which has the ability to solve up to a 36 order polynomial, DRAW which outputs the plot to the CALCOMP plotter, and BCNV which converts numeric symbols to alpha-numeric for use in DRAW.

The program takes the values of zeta and first generates the Chebyshev function of the second kind. It then generates the coefficients for the omega terms in the polynomial expansion of $B_1C_2 - B_2C_1 = 0$. This generation is accomplished by using the coefficients of the alpha and beta terms in the characteristic equation, the value of zeta and the Chebyshev function. The resulting omega coefficients are now input into the subroutine POLRT.

POLRT solves for the roots of the omega polynomial and outputs the results to the program. These roots are now examined for positive real values. If positive real roots are present there is a possibility of singular lines

existing for that value of zeta. If there are not positive real roots, there are no singular lines. This information is printed out and the next value of zeta is examined.

For the positive real roots B_1, B_2, C_1, C_2, D_1, and D_2 of equation (2-27) are formed and used to insure that a singular line does exist. If a singular line does exist then B_1, C_1, and D_1 are used to generate the points along the line by using equation (2-28). These points are now input into the subroutine DRAW and are plotted and labeled.

The printed output includes the value of zeta and omega, the alpha and beta intercept points, the slope, and the label. If there are less than two points generated within the area of the plot there will be no plot of that line. This information is indicated on the printed output and the user can either change the plot scales or use the output line information to construct a singular line plot.

This procedure is repeated for each value of zeta, plotting all the lines on a single plot. Multiple runs for different characteristic equations are possible by using additional data decks. Each equation will have its own output plot.

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SYSIN DD DATA
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     ***********************************************************     C
C     *                                                         *     C
C     *   SINGULAR LINE PROGRAM                                 *     C
C     *   AUGUST 1975                                           *     C
C     *   LT PAUL F. VAN TASSEL, USN                            *     C
C     *                                                         *     C
C     *   THIS PROGRAM IS A DESIGN AID FOR DETERMINING THE      *     C
C     *   EXISTANCE OF SINGULAR LINES FOR A GIVEN VALUE OF ZETA *     C
C     *   FOR A SYSTEM WHOSE CHARACTERISTIC EQUATION HAS COEF-  *     C
C     *   FICIENTS OF THE FORM (B*ALPHA + C*BETA + D) WHERE B,  *     C
C     *   C AND D ARE CONSTANTS AND ALPHA AND BETA RE VARIBLE   *     C
C     *   PARAMETERS.  IF SINGULAR LINES EXIST FOR THE GIVEN    *     C
C     *   VALUES OF ZETA, THE PROGRAM WILL SOLVE FOR OMEGA AND  *     C
C     *   THE ALPHA  AND BETA INTERCEPT POINTS OF THE SINGULAR  *     C
C     *   LINE ON THE ALPHA-BETA PLANE.                         *     C
C     *                                                         *     C
C     *   THE PROGRAM WILL PLOT ON A GRAPH, UP TO 9 INCHES BY   *     C
C     *   15 INCHES, SINGULAR LINES FOR SELECTED VALUES OF ZETA *     C
C     *   BETWEEN ZERO AND ONE.  IN THE EVENT A SINGULAR LINE   *     C
C     *   IS NOT GRAPHED, A PRINTOUT OF THAT LINE WILL INCLUDE  *     C
C     *   THE ALPHA AND BETA INTERCEPTS AND THE SLOPE.  THIS    *     C
C     *   WILL ALLOW THE USER TO EITHER CHANGE THE PLOT SCALES  *     C
C     *   OR TO PLOT THE LINE SEPERATELY.                       *     C
C     *                                                         *     C
C     *   THE FOLLOWING SYMBOLS ARE PERTINANT TO THE PROGRAM    *     C
C     *   AND ARE USED AS INPUT DATA.                           *     C
C     *                                                         *     C
C     *   NUM    THE NUMBER OF CHARACTERISTIC EQUATIONS TO BE   *     C
C     *          EXAMINED.                                      *     C
C     *                                                         *     C
C     *   NO     THE ORDER OF THE CHARACTERISTIC EQUATION TO BE *     C
C     *          EXAMINED.                                      *     C
C     *   NZ     THE NUMBER OF VALUES OF ZETA TO BE CONSIDERED. *     C
C     *          (NOT TO EXCEED 25).                            *     C
C     *   IW     THE WIDTH OF THE OUTPUT GRAPH IN INCHES.  (NOT *     C
C     *          TO EXCEED 9 INCHES)                            *     C
C     *   IH     THE HIEGHT OF THE OUTPUT GRAPH IN INCHES.  (NOT *    C
C     *          TO EXCEED 15 INCHES)                           *     C
C     *   IXUP   THE DISTANCE IN INCHES OF THE ALPHA-AXIS FROM  *     C
C     *          THE BOTTOM OF THE GRAPH.                       *     C
C     *   IYRT   THE DISTANCE IN INCHES OF THE BETA-AXIS FROM   *     C
C     *          THE LEFT OF THE GRAPH.                         *     C
C     *   IG     DETERMINES IF THE GRAPH WILL HAVE A ONE INCH   *     C
C     *          GRID.  IF DESIRED SET IG=1, IF NOT SET IG=0.   *     C
C     *   IPNT   WILL PRINT OUT ALL PLOTTED POINTS.  IF DESIRED *     C
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C      *      SET IPNT=0, IF NOT SET IPNT=1.                          *     C SL 0480
C      *                                                              *     C SL 0490
C      *   ITITLE(1-5)   THE FIRST TITLE LINE ON THE OUTPUT GRAPH.*     C SL 0500
C      *                 MAY CONTAIN UP TO 48 ALPHA-NUMERIC CHAR-  *     C SL 0510
C      *                 ACTERS(INCLUDING BLANKS).                    *     C SL 0520
C      *                                                              *     C SL 0530
C      *   ITITLE(7-12)  THE SECOND TITLE LINE ON THE OUTPUT GRAPH*     C SL 0540
C      *                                                              *     C SL 0550
C      *   D(N)  THE CONSTANT COEFFICIENTS IN THE CHARACTERISTIC     *     C SL 0560
C      *         EQUATION.  READ IN ASCENDING ORDER.                  *     C SL 0570
C      *                                                              *     C SL 0580
C      *   B(N)  THE ALPHA TERM COEFFICIENTS.  READ IN ASCENDING     *     C SL 0590
C      *         ORDER.                                               *     C SL 0600
C      *                                                              *     C SL 0610
C      *   C(N)  THE BETA TERM COEFFICIENTS.  READ IN ASCENDING      *     C SL 0620
C      *         ORDER.                                               *     C SL 0630
C      *                                                              *     C SL 0640
C      *   ZETA(N)   THE VALUES OF ZETA FOR WHICH SINGULAR LINE      *     C SL 0650
C      *             INFORMATION IS DESIRED.  NUMBER MUST CORRE-     *     C SL 0660
C      *             SPOND WITH NZ.                                   *     C SL 0670
C      *                                                              *     C SL 0680
C      *   XSCALE    THE DESIRED SCALE OF THE ALPHA-AXIS.  MUST      *     C SL 0690
C      *             BE ONE SIGNIFICANT FIGURE.                       *     C SL 0700
C      *                                                              *     C SL 0710
C      *   YSCALE    THE DESIRED SCALE OF THE BETA-AXIS.  MUST BE *     C SL 0720
C      *             ONE SIGNIFICANT FIGURE.                          *     C SL 0730
C      *                                                              *     C SL 0740
C      *   THE INPUT DATA FOR THIS PROGRAM IS INSERTED IN THE        *     C SL 0750
C      *   FOLLOWING MANNER.                                          *     C SL 0760
C      *                                                              *     C SL 0770
C      *   CARD 1    NUM - I3 FORMAT                                  *     C SL 0780
C      *   CARD 2    ND,NZ,IW,IH,IXUP,IYRT,IG,IPNT - 8I10 FORMAT     *     C SL 0790
C      *   CARD 3    ITITLE(1-6) - 6A8 FORMAT                         *     C SL 0800
C      *   CARD 4    ITITLE(7-12) - 6A8 FORMAT                        *     C SL 0810
C      *   CARD 5    D(N) - 8E10.5 FORMAT                             *     C SL 0820
C      *   CARD 6    B(N) - 8E10.5 FORMAT                             *     C SL 0830
C      *   CARD 7    C(N) - 8E10.5 FORMAT                             *     C SL 0840
C      *   CARD 8    ZETA(N) - 8E10.5 FORMAT                          *     C SL 0850
C      *   CARD 9    XSCALE - E10.5 FORMAT                            *     C SL 0860
C      *   CARD 10   YSCALE - E10.5 FORMAT                            *     C SL 0870
C      *                                                              *     C SL 0880
C      *   NOTE:                                                      *     C SL 0890
C      *   WITH CARDS 5 THROUGH 8 IT MAY BE NECESSARY TO USE MORE *     C SL 0900
C      *   THEN ONE CARD, DEPENDING ON THE ORDER OF THE SYSTEM       *     C SL 0910
C      *   AND THE NUMBER OF ZETAS TO BE EXAMINED.                    *     C SL 0920
C      *   WHEN THE NUMBER OF CHARACTERISTIC EQUATIONS TO BE         *     C SL 0930
C      *   EXAMINED(NUM) IS GREATER THEN ONE CARDS 2-10 ARE          *     C SL 0940
C      *   REPEATED.                                                  *     C SL 0950
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C      *                                                              *      C
C      ****************************************************************      C
C                                                                            C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      REAL *4 DUM/'1'/,LABEL/'    '/
      REAL *8ITITLE(12)
      DIMENSION B(25), C(25), D(25), ZETA(25), WNS(25), UZ(25), WN(25),
     1B1(25), C1(25), D1(25), IER(4), AG(2), BG(2), LABZ(100), RR(25), R
     2I(25), XWWX(25), B2(25), C2(25), D2(25), WWX(25)
      CALL ERRSET (207,256,-1,1,0,209)
C
C     READ IN THE NUMBER OF DATA SETS.
      READ (5,28) NUM
      DO 24 LONG=1,NUM
      WRITE (6,27)
      READ (5,39) NO,NZ,IW,IH,IXUP,IYRT,IG,IPNT
      NC = NO+1
C
C     INITIALIZE ALL ARRAYS TO ZERO
      DO 1 I=1,25
      B(I) = 0.0
      C(I) = 0.0
      D(I) = 0.0
      ZETA(I) = 0.0
      UZ(I) = 0.0
      WN(I) = 0.0
      B1(I) = 0.0
      C1(I) = 0.0
      D1(I) = 0.0
      B2(I) = 0.0
      C2(I) = 0.0
      D2(I) = 0.0
    1 CONTINUE
C
C     READ AND WRITE THE TITLE FOR THE PLOT
      READ (5,38) (ITITLE(I),I=1,6)
      READ (5,38) (ITITLE(I),I=7,12)
      WRITE (6,40) (ITITLE(I),I=1,6)
      WRITE (6,40) (ITITLE(I),I=7,12)
C
C     READ AND WRITE THE CONSTANT COEFFICIENTS
      WRITE (6,29)
      READ (5,32) (D(N),N=1,NC)
      WRITE (6,33) (D(N),N=1,NC)
C
C     READ AND WRITE THE A-COEFFICIENTS
      WRITE (6,30)
      READ (5,32) (B(N),N=1,NC)
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C
C     READ AND WRITE THE B-COEFFICIENTS
      WRITE (6,33) (B(N),N=1,NC)
      WRITE (6,31)
      READ (5,32) (C(N),N=1,NC)
      WRITE (6,33) (C(N),N=1,NC)
C
C     READ AND WRITE THE VALUES OF ZETA FOR SINGULAR LINES.
      WRITE (6,37)
      READ (5,32) (ZETA(N),N=1,NZ)
      WRITE (6,33) (ZETA(N),N=1,NZ)
C
C     READ AND WRITE THE X-AXIS DIVISION (XSCALE/INCH)
      READ (5,34) XSCALE
      WRITE (6,35) XSCALE
C
C     READ AND WRITE THE Y-AXIS DIVISION (YSCALE/INCH)
      READ (5,34) YSCALE
      WRITE (6,36) YSCALE
      WRITE (6,43)
      DO 23 KK=1,NZ
C
C     GENERATE CHEBYSHEV FUNCTIONS OF THE SECOND KIND FOR A GIVEN ZETA.
      UZ(1) = -1.0
      UZ(2) = 0.0
      NN = NO+2
      DO 2 M=3,NN
      UZ(M) = 2.0*ZETA(KK)*UZ(M-1)-UZ(M-2)
    2 CONTINUE
C
C     GENERATE THE OMEGA(N) COEFFICIENTS FOR THE DETERMINATE OF THE
C     B1C2 - B2C1 MATRIX.
C
      DO 3 I=1,25
      WNS(I) = 0.0
      WWX(I) = 0.0
    3 CONTINUE
      DO 4 I=1,NO
      JJ = NC-I
      DO 4 J=1,JJ
      J2 = (2*I)+(J-2)
      WNS(J2) = (B(I)*C(I+J)-C(I)*B(J+I))*((-1.)**(J+1))*UZ(J+2)+WNS(J2)
      IF (WNS(J2).NE.0.0) JOD=J2
    4 CONTINUE
      NTWO = (2*NO)-1
      DO 5 I=1,NTWO
      J = I+1
    5 WWX(J) = WNS(I)
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      WWX(1) = 0.0
C
C     COMPUTE THE ROOT FOR THE EXPANSION OF (B1C2 - B2C1).
      CALL POLRT (WWX,XWWX,JOD,RR,RI,IER)
      III = 1
      JDO = JOD+1
      DO 6 II=1,JDO
C
C     TEST TO DETERMINE THE EXISTANCE OF REAL ROOTS.
      IF (RR(II).LE.0.0.OR.RI(II).NE.0.0) GO TO 6
      WN(III) = RR(II)
      III = III+1
    6 CONTINUE
      III = III-1
      JT = 0
      MC = 0
      IF (WN(1).NE.0.0) GO TO 7
      WRITE (6,42) ZETA(KK)
      GO TO 23
    7 DO 22 K1=1,III
      IF (JT.LT.2.AND.MC.EQ.1) GO TO 8
      MC = 2
      IF (KK.EQ.1.AND.K1.EQ.1) MC=1
      IF (KK.EQ.NZ.AND.K1.EQ.III) MC=3
      IF (NZ.EQ.1.AND.III.EQ.1) MC=0
    8 CONTINUE
C
C     GENERATE B1, C1, AND D1 FOR EACH VALUE OF OMEGA AT A GIVEN ZETA
      DO 10 K2=1,NC
      K3 = K2+1
      IF ((K2-1).NE.0) GO TO 9
      B1(K1) = -B(K2)
      C1(K1) = -C(K2)
      D1(K1) = -D(K2)
      B2(K1) = 0.0
      C2(K1) = 0.0
      D2(K1) = 0.0
      GO TO 10
    9 ONSQ = (-1.)**(K2-1)
      WNSQ = WN(K1)**(K2-1)
      WNON = ONSQ*WNSQ
      B1(K1) = B(K2)*UZ(K2)*WNON+B1(K1)
      C1(K1) = C(K2)*JZ(K2)*WNON+C1(K1)
      D1(K1) = D(K2)*UZ(K2)*WNON+D1(K1)
      B2(K1) = B(K2)*UZ(K3)*WNON+B2(K1)
      C2(K1) = C(K2)*JZ(K3)*WNON+C2(K1)
      D2(K1) = D(K2)*UZ(K3)*WNON+D2(K1)
   10 CONTINUE
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C
C     TEST TO SEE IF SYSTEM OF EQUATIONS IS INDETERMINATE(B1=B2=C1=C2).
      IF (B1(K1).NE.B2(K1)) GO TO 11
      IF (B1(K1).NE.C1(K1)) GO TO 11
      IF (B1(K1).NE.C2(K1)) GO TO 11
      WRITE (6,44) ZETA(KK)
      GO TO 24
   11 CONTINUE
      IBCK = 0
      IDCK = 0
      IF (C1(K1).NE.0.0.AND.C2(K1).NE.0.0) GO TO 12
      WRITE (6,42) ZETA(KK)
      GO TO 24
   12 IF (B1(K1).EQ.0.0.OR.B2(K1).EQ.0.0) IBCK=1
      IF (D1(K1).EQ.0.0.OR.D2(K1).EQ.0.0) IDCK=1
      IF (IBCK.EQ.0.AND.IDCK.EQ.0) GO TO 14
      IF (B2(K1).NE.B1(K1)) GO TO 13
      IF (D2(K1).NE.D1(K1)) GO TO 13
      GO TO 14
   13 WRITE (6,42) ZETA(KK)
      GO TO 24
   14 CONTINUE
      AMIN = (.25-IYRT)*XSCALE
      AMAX = (IW-IYRT-.25)*XSCALE
      BMIN = YSCALE*(0.25-IXUP)
      BMAX = YSCALE*(IH-IXUP-0.25)
      DELA = (AMAX-AMIN)/300.
C
C     COMPUTE THE X(ALPHA) AND Y(BETA) INTERCEPTS.
      SLOPE = -B1(K1)/C1(K1)
      YINT = -D1(K1)/C1(K1)
      XINT = -(YINT/SLOPE)
      JT = 0
      AG(1) = 0.0
      AG(2) = 0.0
      BG(1) = 0.0
      BG(2) = 0.0
C
C     GENERATE A SINGULAR LINE FOR A GIVEN VALUE OF OMEGA AND ZETA.
      AG(1) = AMIN
   15 BG(1) = -(D1(K1)/C1(K1))-(B1(K1)/C1(K1))*AG(1)
      JT = 1
      IF (BG(1).GE.BMIN.AND.BG(1).LE.BMAX) GO TO 16
      IF (AG(1).GT.AMAX) GO TO 19
      AG(1) = AG(1)+DELA
      GO TO 15
   16 AG(2) = AMAX
      BG(2) = SLOPE*(AMAX-XINT)
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      IF (BG(2).LE.BMAX.AND.BG(2).GE.BMIN) GO TO 18
      IF (SLOPE.LT.0.0) GO TO 17
      BG(2) = BMAX
      AG(2) = (BMAX-YINT)/SLOPE
      GO TO 18
   17 BG(2) = BMIN
      AG(2) = (BMIN-YINT)/SLOPE
   18 JT = 2
   19 CONTINUE
      LABZ(K1) = (K1*10)+KK
C
C     CONVERT THE LABELS FROM NUMERIC TO ALPHA-NUMERIC CHARACTERS.
      CALL BCNV (LABZ(K1),LABBZ,103,4)
      WRITE (6,41) ZETA(KK),WN(K1),XINT,YINT,SLOPE,LABZ(K1)
C
C     PRINT OUT THE POINTS WHICH ARE PLOTTED ON THE GRAPH.
      IF (IPNT.NE.0) GO TO 21
      WRITE (6,25)
      DO 20 I4=1,JT
   20 WRITE (6,26) AG(I4),BG(I4)
      WRITE (6,43)
   21 CONTINUE
C
C     PLOT THE SINGULAR LINES ON THE GRAPH.
      CALL DRAW (JT,AG,BG,MC,0,LABBZ,ITITLE,XSCALE,YSCALE,IXUP,IYRT,2,2,
     1IW,IH,IG,LAST)
   22 CONTINUE
   23 CONTINUE
   24 CONTINUE
C
   25 FORMAT (4X,'POINTS OF THIS SINGULAR LINE WHICH ARE PLOTTED.',//,4X
     1,'ALPHA',11X,'BETA',//)
   26 FORMAT (4X,E12.5,4X,E12.5)
   27 FORMAT ('1    THE INPUT DATA IS:',////)
   28 FORMAT (I3)
   29 FORMAT (//,4X,'CONSTANT COEFFICIENTS IN ASCENDING ORDER',//)
   30 FORMAT (//,4X,'ALPHA COEFFICIENTS IN ASCENDING ORDER',//)
   31 FORMAT (//,4X,'BETA COEFFICIENTS IN ASCENDING ORDER',//)
   32 FORMAT (8E10.5)
   33 FORMAT (4X,8E14.5)
   34 FORMAT (E10.5)
   35 FORMAT (//,4X,'XSCALE:',F10.5,' UNITS PER INCH',//)
   36 FORMAT (//,4X,'YSCALE:',F10.5,' UNITS PER INCH',//)
   37 FORMAT (//,4X,'ZETA VALUES FOR SINGULAR LINES',//)
   38 FORMAT (6A8)
   39 FORMAT (8I10)
   40 FORMAT (4X,6A3)
   41 FORMAT (4X,E12.5,4X,E12.5,6X,E12.5,9X,E12.5,6X,E12.5,6X,I4,//)
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42 FORMAT (4X,E12.5,5X,'NO SINGULAR LINES EXIST FOR THIS VALUE OF ZET SL 3360
  1A.',//)                                                          SL 3370
43 FORMAT ('1     SINGULAR LINE DATA',////,9X,'ZETA',12X,'OMEGA',6X,'X SL 3380
  1-AXIS INTERCEPT',4X,'Y-AXIS INTERCEPT',9X,'SLOPE',9X,'LABEL',//)  SL 3390
44 FORMAT (4X,E12.5,4X,'SYSTEM OF EQUATIONS IS INDETERMINATE.  RANK = SL 3400
  1 0.',//)                                                         SL 3410
   END                                                              SL 3420
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C                               PARAM A                                       PARA
C     ***********************************************************************PARA
C     THIS PROGRAM IS DESIGNED TO AID IN DESIGN AND/OR ANALYSIS OF           PARA
C     CONTROL SYSTEMS OF UP TO ORDER 100 USING PARAMETER PLANE METHODS.      PARA
C     THE CHARACTERISTIC EQUATION HAS VARIABLE PARAMETERS ALFA AND BETA      PARA
C     IN THE FOLLOWING COEFFICIENT FORM:                                     PARA
C                                                                            PARA
C          B*ALFA + C*BETA + D                                               PARA
C                                                                            PARA
C     WHERE B, C, AND D ARE CONSTANTS.                                       PARA
C     A THIRD PARAMETER (E) MAY ALSO BE SPECIFIED AS INDICATED BELOW.        PARA
C     ***********************************************************************PARA
C     THIS PROGRAM WILL PLOT ONE GRAPH UP TO 9 INCHES WIDE BY 15 INCHES      PARA
C     HIGH WITH ALPHA PLOTTED ON THE X-AXIS AND BETA ON THE Y-AXIS.          PARA
C     THE PARAMETER PLANE CURVES OBTAINED ARE THE FOLLOWING TYPE:            PARA
C          1. CONSTANT ZETA CURVES AS A FUNCTION OF OMEGAN.                  PARA
C          2. CONSTANT OMEGAN CURVES AS A FUNCTION OF ZETA.                  PARA
C          3. CONSTANT SIGMA LINES (REAL ROOT LINES).                        PARA
C          4. CONSTANT ZETA*OMEGAN CURVES.                                   PARA
C     ALL CURVES WILL BE PLOTTED ON THE SAME GRAPH.                          PARA
C     ***********************************************************************PARA
C     FIELD DEFINITIONS:                                                     PARA
C          ND - NUMBER OF DECADES SPANNED BY OMEGAN FOR THE CONSTANT ZETAPARA
C               CURVES - SET TO ZERO IF NONE DESIRED. MAXIMUM=100.           PARA
C          NO - ORDER OF THE CHARACTERISTIC EQUATION. MAXIMUM=100.           PARA
C          NZ - NUMBER OF CONSTANT ZETA CURVES DESIRED - SET TO ZERO IF      PARA
C               NONE DESIRED. MAXIMUM=100.                                   PARA
C          NS - NUMBER OF CONSTANT SIGMA(REAL ROOT) CURVES DESIRED - SET     PARA
C               TO ZERO IF NONE DESIRED. MAXIMUM=100.                        PARA
C          NW - NUMBER OF CONSTANT OMEGAN CURVES DESIRED - SET TO ZERO IFPARA
C               NONE DESIRED. MAXIMUM=100.                                   PARA
C          NZW - NUMBER OF CONSTANT ZETA*OMEGAN CURVES DESIRED - SET TO      PARA
C               ZERO IF NONE DESIRED. MAXIMUN=100.                           PARA
C          IXUP - DISTANCE(IN INCHES) THAT XAXIS DISPLACED FROM BOTTOM OFPARA
C               GRAPH.                                                       PARA
C          IYRIGHT - DISTANCE(IN INCHES) THAT YAXIS DISPLACED FROM LEFT      PARA
C               SIDE OF GRAPH.                                               PARA
C          IPRINT - SET TO 1 IF PRINTOUT DESIRED. SET TO 0 IF NO PRINTOUTPARA
C               DESIRED.  IN EITHER CASE IF DATA IS SET UP PROPERLY,PLOTSPARA
C               WILL BE OBTAINED.*****WARNING-JUST 11 VALUES OF ZETA, 20 PARA
C               VALUES OF ZETA*OMEGAN, 2 DECADES OF OMEGAN WILL PRODUCE      PARA
C               OVER 12,000 LINES OF PRINTOUT.  DO NOT USE THIS OPTION       PARA
C               UNLESS ABSOLUTELY NECESSARY.  WHEN USED ON THE NPG 360/67PARA
C               A SPECIAL CARD IS REQUIRED TO GIVE MORE PRINTOUT SPACE.      PARA
C               NORMAL PRINTOUT IS LIMITED TO 3000 LINES.*****               PARA
C          NE - NUMBER OF THIRD PARAMETER VALUES. MAXIMUN=10.                PARA
C          LABZ - LABELS FOR CONSTANT ZETA CURVES.                           PARA
C          LABS - LABELS FOR CONSTANT SIGMA(REAL ROOT) CURVES.               PARA
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C        LABW - LABELS FOR CONSTANT OMEGAN CURVES.                      PARA 490
C        LABZW - LABELS FOR CONSTANT ZETA*OMEGAN CURVES.                PARA 500
C        WN - STARTING VALUE OF OMEGAN FOR THE CONSTANT ZETA CURVES.    PARA 510
C        E - THIRD PARAMETER. MAXIMUN=10.                               PARA 520
C        BJ - ALFA COEFFICIENTS. MAXIMUN=100.                           PARA 530
C        CJ - BETA COEFFICIENTS. MAXIMUM=100.                           PARA 540
C        DJ - CONSTANT COEFFICIENTS. MAXIMUM=100.                       PARA 550
C        XSCALE - SCALE OF XAXIS IN UNITS PER INCH-1 SIGNIFICENT FIGUREPARA 560
C        YSCALE - SCALE OF YAXIS IN UNITS PER INCH-1 SIGNIFICENT FIGUREPARA 570
C        IWIDE AND IHIGH - THE WIDTH AND HIEGHT OF THE GRAPH IN INCHES.PARA 571
C        NRUN - NUMBER OF RUNS.                                         PARA 580
C    **************************************************************PARA 590
C    NOTES:                                                             PARA 600
C        1. ZETA VALUES MUST BE BETWEEN 0.0 AND 1.0                     PARA 610
C        2. CONSTANT SIGMA LINES WILL BE COMPUTED ONLY FOR THOSE VALUESPARA 620
C           OF SIGMA ALONG THE NEGATIVE REAL AXIS IN THE S-PLANE.       PARA 630
C           HOWEVER, THE VALUES OF SIGMA SHOULD BE ENTERED AS POSITIVE PARA 640
C           QUANTITIES.                                                 PARA 650
C        3. FOR LABZ,LABS,LABW,LABZW USE TWO OR MORE CARDS UNTIL NUMBERPARA 660
C           OF FOUR FIELDS EQUALS NZ,NS,NW,NZW RESPECTIVELY.            PARA 670
C        4. FOR ALL VALUES OF ZETA,SIGMA,OMEGAN,ZETA*OMEGAN USE AS MANYPARA 680
C           CARDS(8 VALUES TO A CARD) AS NECESSARY TO INPUT NZ,NS,NW,   PARA 690
C           NZW DATA VALUES RESPECTIVELY.                               PARA 700
C    **************************************************************PARA 710
C    NORMAL DATA CARD SUBMISSION  FOR THIRD PARAMETER NOT SPECIFIED:    PARA 720
C        CARD 0.  NRUN(I10 FORMAT)                                      PARA 730
C        CARD 1.  THE FIRST LINE OF THE GRAPH TITLE (IN COLUMNS 1-48)   PARA 740
C        CARD 2.  THE SECOND LINE OF THE GRAPH TITLE (IN COLUMNS 1-48)  PARA 750
C        CARD 3.  IN 8I10 FORMAT ND  NO  NZ  NS  NW  NZW  IXUP  IYRIGHTPARA 760
C        CARD 4.  IN 2I10 FORMAT IPRINT  NE     (NE=0)                  PARA 770
C        CARD 5.  LABZ (20A4 FORMAT),(USE BLANK CARD IF NZ=0)           PARA 780
C                 (E.G. Z=0.Z=.1Z=.2Z=.3  ETC.)                         PARA 790
C        CARD 6.  LABS (20A4 FORMAT),(USE BLANK CARD IF NS=0)           PARA 800
C        CARD 7.  LABW (20A4 FORMAT),(USE BLANK CARD IF NW=0)           PARA 810
C        CARD 8.  LABZW (20A4 FORMAT),(USE BLANK CARD IF NZW=0)         PARA 820
C        CARD 9.  VALUES OF ZETA FOR CONSTANT ZETA CURVES (8E10.5       PARA 830
C                 FORMAT),(USE BLANK CARD IF NZ=0)                      PARA 840
C        CARD 10. VALUES OF SIGMA FOR CONSTANT SIGMA CURVES (8E10.5     PARA 850
C                 FORMAT),(USE BLANK CARD IF NS=0)                      PARA 860
C        CARD 11. VALUES OF OMEGAN FOR CONSTANT OMEGAN CURVES (8E10.5   PARA 870
C                 FORMAT),(USE BLANK CARD IF NW=0)                      PARA 880
C        CARD 12. VALUES OF ZETA*OMEGAN FOR CONSTANT ZETA*OMEGAN CURVESPARA 890
C                 (8E10.5 FORMAT),(USE BLANK CARD IF NZW=0)             PARA 900
C        CARD 13. CONSTANT COEFFICIENTS (DJ) IN ASCENDING ORDER OF S    PARA 910
C                 (8E10.5 FORMAT)                                       PARA 920
C        CARD 14. ALFA COEFICIENTS (BJ) IN ASCENDING ORDER OF S         PARA 930
C                 (8E10.5 FORMAT)                                       PARA 940
C        CARD 15. BETA COEFFICIENTS (CJ) IN ASCENDING ORDER OF S        PARA 950
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C             (8E10.5 FORMAT)                                                 PARA 960
C        CARD 16. OMEGAN STARTING VALUE (E10.5 FORMAT),(USE BLANK CARD        PARA 970
C                 IF NZ=0)                                                    PARA 980
C        CARD 17. XSCALE AND YSCALE (2E10.5 FORMAT)                           PARA 990
C        CARD 18. IWIDE AND IHIGH (2I5 FORMAT)                                PARA1000
C     ****************************************************************************PARA1010
C     DATA SUBMISSION FOR THIRD PARAMETER SPECIFICATIONS:                     PARA1020
C        CARD 0.  SAME AS CARD 0 IN PREVIOUS SECTION                          PARA1030
C        CARD 1.  SAME AS CARD 1 IN PREVIOUS SECTION                          PARA1040
C        CARD 2.  SAME AS CARD 2 IN PREVIOUS SECTION                          PARA1050
C        CARD 3.  SAME AS CARD 3 IN PREVIOUS SECTION                          PARA1060
C        CARD 4.  SAME AS CARD 4 IN PREVIOUS SECTION EXCEPT NE NOW            PARA1070
C                 INDICATES THE NUMBER OF VALUES OF THE THIRD PARAMETERPARA1080
C        CARD 5 THRU CARD 8  SAME AS CARD 5 THRU CARD 6 OF PREVIOUS           PARA1090
C                 SECTION EXCEPT THE NUMBER OF LABELS REQUIRED IS:            PARA1100
C                      NE*NZ  FOR LABZ                                        PARA1110
C                      NE*NS  FOR LABS                                        PARA1120
C                      NE*NW FOR LABW                                         PARA1130
C                      NE*NZW FOR LABZW                                       PARA1140
C                 LABELS AND VALUES ARE READ FOR ALL VALUES OF NE FOR         PARA1150
C                 EACH LABEL AND VALUE OF THE CURVE DESIGNATED.               PARA1160
C        CARD 9 THRU CARD 12  SAME AS CARD 9 THRU CARD 12 IN PREVIOUS         PARA1170
C                 SECTION EXCEPT WITH VALUES INDICATED IN CARDS 5             PARA1180
C                 THRU 8 OF THIS SECTION.                                     PARA1190
C        CARD 13. VALUES OF THIRD PARAMETER (E),(8E10.5 FORMAT)               PARA1200
C        CARD 14. SAME AS CARD 16 IN PREVIOUS SECTION                         PARA1210
C        CARD 15. SAME AS CARD 17 IN PREVIOUS SECTION                         PARA1220
C        CARD 16. SAME AS CARD 18 IN PREVIOUS SECTION                         PARA1230
C     THE COEFFICIENTS OF THE CHARACTERISTIC EQUATION MUST BE ENTERED INPARA1240
C     SUBROUTINE COEF.  THE THIRD PARAMETER E MAY APPEAR LINEARILY OR         PARA1250
C     NON-LINEARILY IN ANY OF THE COEFFICIENTS. FOR EXAMPLE, GIVEN THE        PARA1260
C     CHARACTERISTIC EQUATION:                                                PARA1270
C        (BJ(4)*ALFA+CJ(4)*BETA+DJ(4))*S**3                                   PARA1280
C             +(BJ(3)*ALFA+CJ(3)*BETA+DJ(3))*S**2                             PARA1290
C                  +(BJ(2)*ALFA+CJ(2)*BETA+DJ(2))*S                           PARA1300
C                       +BJ(1)*ALFA+CJ(1)*BETA+DJ(1)=0                        PARA1310
C     THE COEFFICIENTS OF THE ABOVE EQUATION CAN BE ENTERED IN COEF AS        PARA1320
C     FOLLOWS:                                                                PARA1330
C                                                                             PARA1340
C        COMMON E,BJ,CJ,DJ                                                    PARA1350
C       BJ(4)=1.                                                              PARA1360
C       CJ(4)=0.                                                              PARA1370
C       DJ(4)=11.                                                             PARA1380
C       BJ(3)=11.                                                             PARA1390
C       CJ(3)=E                                                               PARA1400
C       DJ(3)=10.                                                             PARA1410
C       BJ(2)=10.+E                                                           PARA1420
C       CJ(2)=286.+28.6*E                                                     PARA1430
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C         DJ(2)=0.                                                        PARA1440
C         BJ(1)=2.+E                                                      PARA1450
C         CJ(1)=1.                                                        PARA1460
C         DJ(1)=3.                                                        PARA1470
C         RETURN                                                          PARA1480
C     ******************************************************************PARA1490
C     NOTE:                                                               PARA1500
C         ALL THIRD PARAMETER CURVES WILL BE OUTPUT ON THE SAME GRAPH.    PARA1510
C     MULTIPLE RUNS ARE SUBMITTED WITH ALL NEW DATA CARDS FOR EACH RUN    PARA1520
C     AND THE NUMBER OF RUNS INDICATED ON CARD 0 IN THE NRUN FIELD.       PARA1530
C     RUN TIME FOR A THIRD ORDER EQUATION WITH NZ=11, NZW=20, NE=1 FOR    PARA1540
C     5 RUNS IS APPROXIMATELY 1 MINUTE.  CORE REQUIREMENTS DO NOT         PARA1550
C     EXCEED 150K.                                                        PARA1560
C     ******************************************************************PARA1570
C                                                                         PARA1580
      REAL*4 DUM/1H1/,LABEL/4H    /                                       PARA1590
      REAL *8ITITLE(12)                                                   PARA1600
      DIMENSION A(350), B(350), ZETA(100), LABZ(100,10), SIGMA(100), LABPARA1610
     1S(100,10), W(100), LABW(100,10), ZW(100), LABZW(100,10), AG(350),   PARA1620
     2BG(350), EJ(100), BJ(100), CJ(100), DJ(100)                         PARA1630
      COMMON E,BJ,CJ,DJ                                                   PARA1640
      READ (5,109) NRUN                                                   PARA1650
C                                                                         PARA1660
      DO 104 IRUN=1,NRUN                                                  PARA1670
      WRITE (6,105)                                                       PARA1680
      READ (5,106) (ITITLE(I),I=1,6)                                      PARA1690
      READ (5,106) (ITITLE(I),I=7,12)                                     PARA1700
      WRITE (6,107) (ITITLE(I),I=1,6)                                     PARA1710
      WRITE (6,107) (ITITLE(I),I=7,12)                                    PARA1720
      WRITE (6,108)                                                       PARA1730
      READ (5,109) ND,NO,NZ,NS,NW,NZW,IXUP,IYRITE                         PARA1740
      NC = NO+1                                                           PARA1750
      WRITE (6,109) ND,NO,NZ,NS,NW,NZW,IXUP,IYRITE                        PARA1760
      WRITE (6,110)                                                       PARA1770
      READ (5,111) IPRINT,NE                                              PARA1780
      WRITE (6,111) IPRINT,NE                                             PARA1790
      IF (NE) 1,1,2                                                       PARA1800
    1 NEE = NE+1                                                          PARA1810
      GO TO 3                                                             PARA1820
    2 NEE = NE                                                            PARA1830
    3 AIXUP = IXUP                                                        PARA1840
      AIYRIT = IYRITE                                                     PARA1850
      WRITE (6,112)                                                       PARA1860
      READ (5,113) ((LABZ(M,N),M=1,NZ),N=1,NEE)                           PARA1870
      WRITE (6,114) ((LABZ(M,N),M=1,NZ),N=1,NEE)                          PARA1880
      WRITE (6,115)                                                       PARA1890
      READ (5,113) ((LABS(M,N),M=1,NS),N=1,NEE)                           PARA1900
      WRITE (6,114) ((LABS(M,N),M=1,NS),N=1,NEE)                          PARA1910
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      WRITE (6,116)                                                     PARA1920
      READ (5,113) ((LABW(M,N),M=1,NW),N=1,NEE)                         PARA1930
      WRITE (6,114) ((LABW(M,N),M=1,NW),N=1,NEE)                        PARA1940
      WRITE (6,117)                                                     PARA1950
      READ (5,113) ((LABZW(M,N),M=1,NZW),N=1,NEE)                       PARA1960
      WRITE (6,114) ((LABZW(M,N),M=1,NZW),N=1,NEE)                      PARA1970
      WRITE (6,120)                                                     PARA1980
      READ (5,118) (ZETA(M),M=1,NZ)                                     PARA1990
      WRITE (6,119) (ZETA(M),M=1,NZ)                                    PARA2000
      WRITE (6,121)                                                     PARA2010
      READ (5,118) (SIGMA(M),M=1,NS)                                    PARA2020
      WRITE (6,119) (SIGMA(M),M=1,NS)                                   PARA2030
      WRITE (6,122)                                                     PARA2040
      READ (5,118) (W(M),M=1,NW)                                        PARA2050
      WRITE (6,119) (W(M),M=1,NW)                                       PARA2060
      WRITE (6,123)                                                     PARA2070
      READ (5,118) (ZW(M),M=1,NZW)                                      PARA2080
      WRITE (6,119) (ZW(M),M=1,NZW)                                     PARA2090
      IF (NE) 4,4,5                                                     PARA2100
    4 WRITE (6,124)                                                     PARA2110
      READ (5,118) (DJ(N),N=1,NC)                                       PARA2120
      WRITE (6,119) (DJ(N),N=1,NC)                                      PARA2130
      WRITE (6,125)                                                     PARA2140
      READ (5,118) (BJ(N),N=1,NC)                                       PARA2150
      WRITE (6,119) (BJ(N),N=1,NC)                                      PARA2160
      WRITE (6,126)                                                     PARA2170
      READ (5,118) (CJ(N),N=1,NC)                                       PARA2180
      WRITE (6,119) (CJ(N),N=1,NC)                                      PARA2190
      GO TO 6                                                           PARA2200
    5 WRITE (6,127)                                                     PARA2210
      READ (5,118) (EJ(N),N=1,NE)                                       PARA2220
      WRITE (6,119) (EJ(N),N=1,NE)                                      PARA2230
    6 CONTINUE                                                          PARA2240
      WRITE (6,128)                                                     PARA2250
      READ (5,129) WN                                                   PARA2260
      WRITE (6,130) WN                                                  PARA2270
      WRITE (6,131)                                                     PARA2280
      READ (5,129) XSCALE, YSCALE                                       PARA2290
      WRITE (6,130) XSCALE, YSCALE                                      PARA2300
      WRITE (6,132)                                                     PARA2310
      READ (5,143) IWIDE, IHIGH                                         PARA2320
      WRITE (6,144) IWIDE, IHIGH                                        PARA2330
      ROG = AIYRIT-0.25                                                 PARA2340
      DAV = AIXUP-0.25                                                  PARA2350
      AROG = -ROG*XSCALE                                                PARA2360
      ADAV = -DAV*YSCALE                                                PARA2370
      ROGE = IWIDE-0.25-AIYRIT                                          PARA2380
      DAVE = IHIGH-0.25-AIXUP                                           PARA2390
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      AROGE = ROGE*XSCALE                                                PARA2400
      ADAVE = DAVE*YSCALE                                                PARA2410
      FRAN = AROG                                                        PARA2420
      CHEK = ADAV                                                        PARA2430
      IF (NZ) 18,18,7                                                    PARA2440
    7 GO TO (8,9,10,11,12,13,14,15,16,17), ND                            PARA2450
    8 G = 1.0076                                                         PARA2460
      GO TO 18                                                           PARA2470
    9 G = 1.016                                                          PARA2480
      GO TO 18                                                           PARA2490
   10 G = 1.0245                                                         PARA2500
      GO TO 18                                                           PARA2510
   11 G = 1.0312                                                         PARA2520
      GO TO 18                                                           PARA2530
   12 G = 1.0394                                                         PARA2540
      GO TO 18                                                           PARA2550
   13 G = 1.0483                                                         PARA2560
      GO TO 18                                                           PARA2570
   14 G = 1.0568                                                         PARA2580
      GO TO 18                                                           PARA2590
   15 G = 1.0633                                                         PARA2600
      GO TO 18                                                           PARA2610
   16 G = 1.071                                                          PARA2620
      GO TO 18                                                           PARA2630
   17 G = 1.078                                                          PARA2640
   18 CONTINUE                                                           PARA2650
      AG(1) = 0.0                                                        PARA2660
      BG(1) = 0.0                                                        PARA2670
      AG(2) = XSCALE                                                     PARA2680
      BG(2) = 0.0                                                        PARA2690
      CALL DRAW (2,AG,BG,1,0,LABEL,ITITLE,XSCALE,YSCALE,IXUP,IYRITE,2,2,PARA2700
     1IWIDE,IHIGH,1,LAST)                                                PARA2710
      IF (NZ) 38,38,19                                                   PARA2720
   19 IF (IPRINT) 21,21,20                                               PARA2730
   20 WRITE (6,133)                                                      PARA2740
      WRITE (6,134)                                                      PARA2750
   21 MOD = 2                                                            PARA2760
C                                                                        PARA2770
      DO 37 ME=1,NEE                                                     PARA2780
      E = EJ(ME)                                                         PARA2790
      IF (NE) 23,23,22                                                   PARA2800
   22 CALL COEF                                                          PARA2810
C                                                                        PARA2820
   23 DO 36 M=1,NZ                                                       PARA2830
      J = 0                                                              PARA2840
      JG = 0                                                             PARA2850
      WNA = WN                                                           PARA2860
C                                                                        PARA2870
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      DO 35 L=1,300                                                     PARA2880
      D1 = 0.0                                                          PARA2890
      D2 = 0.0                                                          PARA2900
      C1 = 0.0                                                          PARA2910
      C2 = 0.0                                                          PARA2920
      B1 = 0.0                                                          PARA2930
      B2 = 0.0                                                          PARA2940
C                                                                       PARA2950
      DO 26 N=1,NC                                                      PARA2960
      K = N-1                                                           PARA2970
      IF (K) 25,24,25                                                   PARA2980
   24 U = 0.0                                                           PARA2990
      U1 = -1.0                                                         PARA3000
   25 U2 = 2.0*ZETA(M)*U-U1                                             PARA3010
      D1 = (-1.0)**K*DJ(N)*WNA**K*U1+D1                                 PARA3020
      D2 = (-1.0)**K*DJ(N)*WNA**K*U+D2                                  PARA3030
      C1 = (-1.0)**K*CJ(N)*WNA**K*U1+C1                                 PARA3040
      C2 = (-1.0)**K*CJ(N)*WNA**K*U+C2                                  PARA3050
      B1 = (-1.0)**K*BJ(N)*WNA**K*U1+B1                                 PARA3060
      B2 = (-1.0)**K*BJ(N)*WNA**K*U+B2                                  PARA3070
      U1 = U                                                            PARA3080
   26 U = U2                                                            PARA3090
C                                                                       PARA3100
      Z = 1.0E-60                                                       PARA3110
      IF (ABS(B1*C2-B2*C1)-Z) 27,27,28                                  PARA3120
   27 GO TO 35                                                          PARA3130
   28 J = J+1                                                           PARA3140
      A(J) = (C1*D2-C2*D1)/(B1*C2-B2*C1)                                PARA3150
      B(J) = (B2*D1-B1*D2)/(B1*C2-B2*C1)                                PARA3160
      IF (IPRINT) 30,30,29                                              PARA3170
   29 WRITE (6,135) A(J),B(J),WNA,ZETA(M),E                             PARA3180
   30 IF (FRAN-A(J)) 31,31,35                                           PARA3190
   31 IF (A(J)-AROGE) 32,32,35                                          PARA3200
   32 IF (CHEK-B(J)) 33,33,35                                           PARA3210
   33 IF (B(J)-ADAVE) 34,34,35                                          PARA3220
   34 JG = JG+1                                                         PARA3230
      AG(JG) = A(J)                                                     PARA3240
      BG(JG) = B(J)                                                     PARA3250
   35 WNA = G*WNA                                                       PARA3260
C                                                                       PARA3270
      CALL DRAW (JG,AG,BG,MOD,0,LABZ(M,ME),ITITLE,XSCALE,YSCALE,IXUP,IYRPARA3280
     1ITE,2,2,IWIDE,IHIGH,1,LAST)                                       PARA3290
   36 CONTINUE                                                          PARA3300
C                                                                       PARA3310
   37 CONTINUE                                                          PARA3320
C                                                                       PARA3330
   38 IF (NS) 64,64,39                                                  PARA3340
   39 IF (IPRINT) 41,41,40                                              PARA3350
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```
   40 WRITE (6,136)
      WRITE (6,137)
C
   41 DO 65 ME=1,NEE
      E = EJ(ME)
      IF (NE) 43,43,42
   42 CALL COEF
C
   43 DO 64 M=1,NS
      DD = 0.0
      CC = 0.0
      BB = 0.0
C
      DO 44 N=1,NC
      K = N-1
      DUMMY5 = 0.0
      IF (SIGMA(M).NE.0.0) DUMMY5=SIGMA(M)**K
      DD = (-1.0)**K*DJ(N)*DUMMY5+DD
      CC = (-1.0)**K*CJ(N)*DUMMY5+CC
      BB = (-1.0)**K*BJ(N)*DUMMY5+BB
   44 CONTINUE
C
      J = 1
      A(J) = -DD/BB
      B(J) = 0.0
      IF (IPRINT) 46,46,45
   45 WRITE (6,138) A(J),B(J),SIGMA(M),E
   46 IF (AROG-A(J)) 47,47,49
   47 IF (A(J)-AROGE) 48,48,49
   48 J = J+1
   49 A(J) = 0.0
      B(J) = -DD/CC
      IF (IPRINT) 51,51,50
   50 WRITE (6,138) A(J),B(J),SIGMA(M),E
   51 IF (ADAV-B(J)) 52,52,54
   52 IF (B(J)-ADAVE) 53,53,54
   53 J = J+1
   54 B(J) = (IHIGH-0.25-AIXUP)*YSCALE
      A(J) = (-CC*B(J)-DD)/BB
      IF (IPRINT) 56,56,55
   55 WRITE (6,138) A(J),B(J),SIGMA(M),E
   56 IF (AROG-A(J)) 57,57,58
   57 IF (A(J)-AROGE) 63,63,58
   58 A(J) = (IWIDE-0.25-AIYRIT)*XSCALE
      B(J) = (-BB*A(J)-DD)/CC
      IF (IPRINT) 60,60,59
   59 WRITE (6,138) A(J),B(J),SIGMA(M),E
   60 IF (ADAV-B(J)) 61,61,62
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      61 IF (B(J)-ADAVE) 63,63,62                                        PARA3840
      62 J = J-1                                                         PARA3850
      63 CALL DRAW (J,A,B,2,0,LABS(M,ME),ITITLE,XSCALE,YSCALE,IXUP,IYRITE,2PARA3860
        12,IWIDE,IHIGH,1,LAST)                                           PARA3870
      64 CONTINUE                                                        PARA3880
C                                                                        PARA3890
      65 CONTINUE                                                        PARA3900
C                                                                        PARA3910
         IF (NZW) 84,84,66                                               PARA3920
      66 IF (IPRINT) 68,68,67                                            PARA3930
      67 WRITE (6,139)                                                   PARA3940
         WRITE (6,140)                                                   PARA3950
C                                                                        PARA3960
      68 DO 83 ME=1,NEE                                                  PARA3970
         E = EJ(ME)                                                      PARA3980
         IF (NE) 70,70,69                                                PARA3990
      69 CALL COEF                                                       PARA4000
C                                                                        PARA4010
      70 DO 82 M=1,NZW                                                   PARA4020
         J = 0                                                           PARA4030
         JG = 0                                                          PARA4040
         AZETA = .00333                                                  PARA4050
C                                                                        PARA4060
         DO 81 L=1,299                                                   PARA4070
         WN = ZW(M)/AZETA                                                PARA4080
         D1 = 0.0                                                        PARA4090
         D2 = 0.0                                                        PARA4100
         C1 = 0.0                                                        PARA4110
         C2 = 0.0                                                        PARA4120
         B1 = 0.0                                                        PARA4130
         B2 = 0.0                                                        PARA4140
C                                                                        PARA4150
         DO 73 N=1,NC                                                    PARA4160
         K = N-1                                                         PARA4170
         IF (K) 72,71,72                                                 PARA4180
      71 Q1 = 0.0                                                        PARA4190
         Q = -1.0/WN**2                                                  PARA4200
      72 D2 = DJ(N)*Q1+D2                                                PARA4210
         C2 = CJ(N)*Q1+C2                                                PARA4220
         B2 = BJ(N)*Q1+B2                                                PARA4230
         D1 = DJ(N)*Q+D1                                                 PARA4240
         C1 = CJ(N)*Q+C1                                                 PARA4250
         B1 = BJ(N)*Q+B1                                                 PARA4260
         Q2 = -2.0*ZW(M)*Q1-WN**2*Q                                      PARA4270
         Q = Q1                                                          PARA4280
      73 Q1 = Q2                                                         PARA4290
C                                                                        PARA4300
         IF (ABS(B1*C2-B2*C1)-Z) 81,81,74                                PARA4310
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   74 J = J+1                                                            PARA4320
      A(J) = (C1*D2-C2*D1)/(B1*C2-B2*C1)                                 PARA4330
      B(J) = (B2*D1-B1*D2)/(B1*C2-B2*C1)                                 PARA4340
      IF (IPRINT) 76,76,75                                               PARA4350
   75 WRITE (6,138) A(J),B(J),ZW(M),E                                    PARA4360
   76 IF (FRAN-A(J)) 77,77,81                                            PARA4370
   77 IF (A(J)-ARDGE) 78,78,81                                           PARA4380
   78 IF (CHEK-B(J)) 79,79,81                                            PARA4390
   79 IF (B(J)-ADAVE) 80,80,81                                           PARA4400
   80 JG = JG+1                                                          PARA4410
      AG(JG) = A(J)                                                      PARA4420
      BG(JG) = B(J)                                                      PARA4430
   81 AZETA = AZETA+.00333                                               PARA4440
C                                                                        PARA4450
      CALL DRAW (JG,AG,BG,2,0,LABZW(M,ME),ITITLE,XSCALE,YSCALE,IXUP,IYRIPARA4460
     1TE,2,2,IWIDE,IHIGH,1,LAST)                                         PARA4470
   82 CONTINUE                                                           PARA4480
C                                                                        PARA4490
   83 CONTINUE                                                           PARA4500
C                                                                        PARA4510
   84 IF (NW) 103,103,85                                                 PARA4520
   85 IF (IPRINT) 87,87,86                                               PARA4530
   86 WRITE (6,141)                                                      PARA4540
      WRITE (6,142)                                                      PARA4550
C                                                                        PARA4560
   87 DO 102 ME=1,NEE                                                    PARA4570
      E = EJ(ME)                                                         PARA4580
      IF (NE) 89,89,88                                                   PARA4590
   88 CALL COEF                                                          PARA4600
C                                                                        PARA4610
   89 DO 101 M=1,NW                                                      PARA4620
      J = 0                                                              PARA4630
      JG = 0                                                             PARA4640
      AZETA = 0.0                                                        PARA4650
C                                                                        PARA4660
      DO 100 L=1,300                                                     PARA4670
      D1 = 0.0                                                           PARA4680
      D2 = 0.0                                                           PARA4690
      C1 = 0.0                                                           PARA4700
      C2 = 0.0                                                           PARA4710
      B1 = 0.0                                                           PARA4720
      B2 = 0.0                                                           PARA4730
C                                                                        PARA4740
      DO 92 N=1,NC                                                       PARA4750
      K = N-1                                                            PARA4760
      IF (K) 91,90,91                                                    PARA4770
   90 U = 0.0                                                            PARA4780
      U1 = -1.0                                                          PARA4790
```

```
   91 U2 = 2.0*AZETA*U-U1
      D1 = (-1.0)**K*DJ(N)*W(M)**K*U1+D1
      D2 = (-1.0)**K*DJ(N)*W(M)**K*U+D2
      C1 = (-1.0)**K*CJ(N)*W(M)**K*U1+C1
      C2 = (-1.0)**K*CJ(N)*W(M)**K*U+C2
      B1 = (-1.0)**K*BJ(N)*W(M)**K*U1+B1
      B2 = (-1.0)**K*BJ(N)*W(M)**K*U+B2
      U1 = U
   92 U = U2
C
      IF (ABS(B1*C2-B2*C1)-Z) 100,100,93
   93 J = J+1
      A(J) = (C1*D2-C2*D1)/(B1*C2-B2*C1)
      B(J) = (B2*D1-B1*D2)/(B1*C2-B2*C1)
      IF (IPRINT) 95,95,94
   94 WRITE (6,135) A(J),B(J),W(M),AZETA,E
   95 IF (FRAN-A(J)) 96,96,100
   96 IF (A(J)-AROGE) 97,97,100
   97 IF (CHEK-B(J)) 98,98,100
   98 IF (B(J)-ADAVE) 99,99,100
   99 JG = JG+1
      AG(JG) = A(J)
      BG(JG) = B(J)
  100 AZETA = AZETA+.00333
C
  101 CALL DRAW (JG,AG,BG,MOD,0,LABW(M,ME),ITITLE,XSCALE,YSCALE,IXUP,IYR
     1ITE,2,2,IWIDE,IHIGH,1,LAST)
C
  102 CONTINUE
C
  103 AG(1) = 0.0
      BG(1) = 0.0
      AG(2) = XSCALE
      BG(2) = 0.0
      CALL DRAW (2,AG,BG,3,0,LABEL,ITITLE,XSCALE,YSCALE,IXUP,IYRITE,2,2,
     1IWIDE,IHIGH,1,LAST)
  104 CONTINUE
C
      STOP
C
  105 FORMAT (1H1,17HTHE INPUT DATA IS,////)
  106 FORMAT (6A8)
  107 FORMAT (3X,6A8)
  108 FORMAT (//,8X,2HND,8X,2HNO,8X,2HNZ,8X,2HNS,8X,2HNW,7X,3HNZW,6X,4HI
     1XUP,3X,7HIYRIGHT,//)
  109 FORMAT (8I10)
  110 FORMAT (//,4X,6HIPRINT,8X,2HNE,//)
  111 FORMAT (2I10)
```

```
112 FORMAT (//,10X,' LABZ ',//)                                          PARA5280
113 FORMAT (20A4)                                                        PARA5290
114 FORMAT (3X,20A4)                                                     PARA5300
115 FORMAT (//,10X,' LABS ',//)                                          PARA5310
116 FORMAT (//,10X,' LABW ',//)                                          PARA5320
117 FORMAT (//,10X,' LABZW',//)                                          PARA5330
118 FORMAT (8E10.5)                                                      PARA5340
119 FORMAT (3X,8E10.5)                                                   PARA5350
120 FORMAT (//,10X,' ZETA ',//)                                          PARA5360
121 FORMAT (//,10X,'SIGMA ',//)                                          PARA5370
122 FORMAT (//,10X,' W    ',//)                                          PARA5380
123 FORMAT (//,10X,'ZW    ',//)                                          PARA5390
124 FORMAT (//,10X,'CONSTANT COEFFICIENTS IN ASCENDING ORDER',//)        PARA5400
125 FORMAT (//,10X,'ALPHA    COEFFICIENTS IN ASCENDING ORDER',//)        PARA5410
126 FORMAT (//,10X,'BETA     COEFFICIENTS IN ASCENDING ORDER',//)        PARA5420
127 FORMAT (//,10X,'VALUES OF THE THIRD PARAMETER               ',//)    PARA5430
128 FORMAT (//,10X,'INITIAL VALUE OF OMEGA                      ',//)    PARA5440
129 FORMAT (2E10.5)                                                      PARA5450
130 FORMAT (5X,E10.5,8X,E10.5,/)                                         PARA5460
131 FORMAT (/,7X,'XSCALE',10X,'YSCALE',/)                                PARA5470
132 FORMAT (/,4X,'PLOT WIDTH',4X,'PLOT HIEGHT',/)                        PARA5480
133 FORMAT (1H1,10X,'CONSTANT ZETA CURVES                ',//)           PARA5490
134 FORMAT (//,15X,5HALPHA,16X,4HBETA,15X,5HOMEGA,16X,4HZETA,5X,15HTHIPARA5500
   1RD PARAMETER,//)                                                     PARA5510
135 FORMAT (5E20.5)                                                      PARA5520
136 FORMAT (1H1,10X,'CONSTANT SIGMA CURVES               ',//)           PARA5530
137 FORMAT (15X,5HALPHA,16X,4HBETA,15X,5HSIGMA,5X,15HTHIRD PARAMETER,/PARA5540
   1/)                                                                   PARA5550
138 FORMAT (4E20.5)                                                      PARA5560
139 FORMAT (1H1,25HCONSTANT ZETA-OMEGA CURVES,//)                        PARA5570
140 FORMAT (15X,5HALPHA,16X,4HBETA,10X,10HZETA-OMEGA,5X,15HTHIRD PARAMPARA5580
   1ETER,//)                                                             PARA5590
141 FORMAT (1H1,21HCONSTANT OMEGA CURVES,//)                             PARA5600
142 FORMAT (15X,5HALPHA,16X,4HBETA,15X,5HOMEGA,15X,5HAZETA,5X,15HTHIRDPARA5610
   1 PARAMETER,//)                                                       PARA5620
143 FORMAT (2I5)                                                         PARA5630
144 FORMAT (/,5X,I1,' INCHES',6X,I2,' INCHES',/)                         PARA5640
    END                                                                  PARA5650
    SUBROUTINE COEF                                                      COEF  10
    DIMENSION BJ(100), CJ(100), DJ(100)                                  COEF  20
    COMMON E,BJ,CJ,DJ                                                    COEF  30
    RETURN                                                               COEF  40
    END                                                                  COEF  50
```

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